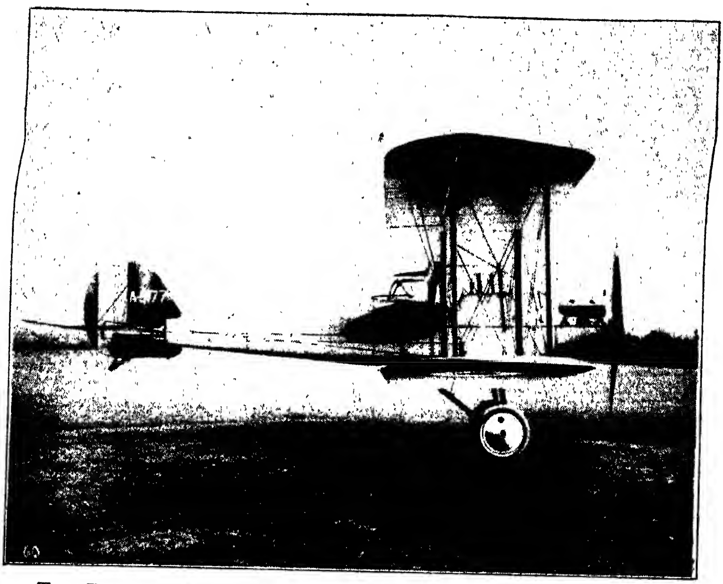
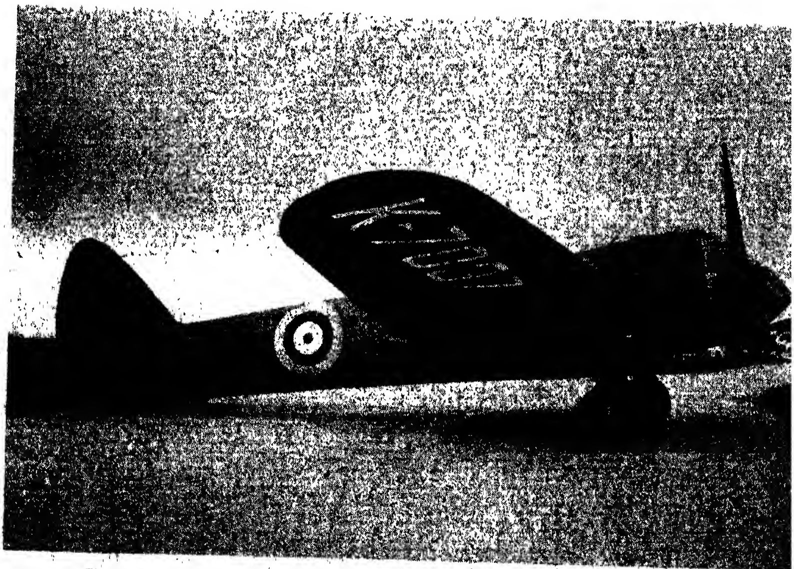
STRUCTURES



THE BRISTOL M.R.1, THE FIRST BRITISH ALL-METAL AIRCRAFT TO TAKE THE AIR



BRISTOL "BLENHEIM" BOMBER, A MODERN METAL AEROPLANE

Frontispiece

An Introduction *to* Aeronautical Engineering

FOR STUDENTS ENGAGED IN ALL BRANCHES OF AERONAUTICAL WORK

Vol. II
STRUCTURES

BY

J. D. HADDON, B.Sc.

Fellow of the Royal Aeronautical Society
Author of "A Simple Study of Flight"

FIFTH AND REVISED EDITION

LONDON
SIR ISAAC PITMAN & SONS, LTD.
1945

FIRST EDITION 1931
SECOND AND REVISED EDITION 1934
REPRINTED 1936
THIRD AND REVISED EDITION 1938
REPRINTED 1939
FOURTH AND REVISED EDITION 1940
FIFTH AND REVISED EDITION 1942
REPRINTED 1945

SIR ISAAC PITMAN & SONS, LTD.
PITMAN HOUSE, PARKER STREET, KINGSWAY, LONDON, W.C.2
THE PITMAN PRESS, BATH
PITMAN HOUSE, LITTLE COLLINS STREET, MELBOURNE
UNITEERS BUILDING, RIVER VALLEY ROAD, SINGAPORE
27 BECKETTS BUILDINGS, PRESIDENT STREET, JOHANNESBURG

ASSOCIATED COMPANIES
PITMAN PUBLISHING CORPORATION
2 WEST 45TH STREET, NEW YORK
205 WEST MONROE STREET, CHICAGO
SIR ISAAC PITMAN & SONS (CANADA), LTD.
(INCORPORATING THE COMMERCIAL TEXT BOOK COMPANY)
PITMAN HOUSE, 381–383 CHURCH STREET, TORONTO

BOOK
PRODUCTION
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STANDARD

THE PAPER AND BINDING OF
THIS BOOK CONFORM TO THE
AUTHORIZED ECONOMY STANDARDS

MADE IN GREAT BRITAIN AT THE PITMAN PRESS, BATH
D5—(A.9001)

INTRODUCTORY

THIS series is intended for the use of all those who are engaged in practical aeronautical engineering, and who feel the need of at least an elementary knowledge of the theory underlying their practical work. For this reason it is hoped that it may appeal equally to draughtsmen, apprentices, pilots, and students at technical schools who are desirous of entering some kind of aeronautical work.

The authors have aimed at producing books which, while avoiding the use of higher mathematics, are at the same time correct, up-to-date, and as free as possible from the technical errors so often found in books which attempt to be "popular" at the expense of everything else. To what extent the authors have succeeded in their object the reader alone can judge.

Except in the methods of putting the subject before the reader, no claim to originality can be made in books of this kind, and the authors acknowledge their indebtedness to all the standard works on aeronautics.

The present series consists of—

Vols. I. Mechanics of Flight, by A. C. Kermode.
II. Structures, by J. D. Haddon.
III. Properties and Strength of Materials, by J. D. Haddon.

Vol. I outlines the principles which maintain an aeroplane in flight, Vol. II is confined more to the internal structural problems of aeroplane design, while Vol. III deals with the strength of the members and the materials used in them.

Subsequent volumes will deal with other subjects directly connected with aeronautical work.

The authors will welcome criticism of any kind.

A. C. K.
J. D. H.

PREFACE TO THIS VOLUME

THE general purposes of the series have already been given.

This volume is devoted to internal structural problems. I have endeavoured to give some of the principles underlying aeroplane design subject to the limitations imposed by the general preface.

For instance, in dealing with spars no attempt has been made to analyse specific cases, as these would involve the use of advanced mathematics; similarly, I have avoided any analysis of the torsional loads in the fuselage. It is hoped, however, that in such instances sufficient information has been given to enable the student at least to understand the principles of design and construction.

The reader is advised to read the corresponding volume on "Mechanics of Flight" in the same series, in order that he may fully understand the subject. It will be found that any knowledge in that subject which has been assumed in this book has been fully covered by the companion volume.

In a work of this kind I am obviously indebted to the help of many others, but I should particularly like to thank those aircraft firms who have provided photographs and other information, the proprietors of *Flight* for permission to reproduce many of their sketches, and Mr. Marcus Langley, who has allowed me to use illustrations from his book, *Metal Aircraft Construction*.

J. D. HADDON.

PREFACE TO SECOND EDITION

In publishing this second edition I desire to thank all those who have given constructive criticism of the first edition.

I am fortunate in teaching students who are studying from this book, and have thus been able to discover any parts that required elucidating. A number of minor alterations have been made as a result. Additional matter has been added, of which the most important is Undercarriage Strength.

The type is clearer and larger in this edition, and this I hope will mean less fatiguing study.

J. D. H.

PREFACE TO THIRD, FOURTH AND FIFTH EDITIONS

Several alterations and additions have been made in these editions, including Proof and Ultimate Factor, Stressed Skin and Geodetic Construction, Flexibility, Retractable Undercarriages, a fuller treatment of Elastic Instability, and an introduction to the Principle of Least Work.

I would again thank those who have pointed out where improvements could be made.

J. D. H.

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STRUCTURES

CHAPTER I

MECHANICS

BEFORE aeronautics can be efficiently studied, the student must have a thorough knowledge of the principles of mechanics.

The object of this chapter is to remind readers of the main principles required. For those who are unfamiliar with mechanics we recommend them to first study any good book on the subject.

Force

The effect of force is to tend to change the state of rest or motion of a body.

If two equal and opposite forces be applied to a body in the same straight line, they will balance each other, and the body is said to be in a state of equilibrium, i.e. it will continue in its state of rest or steady motion.

Most bodies at rest have two equal and opposite forces acting on them; their weight acting vertically downwards and the reaction of their support vertically upwards.

If a balloon is at rest in still air, the weight is equal and opposite to the buoyancy force. If it is rising at constant speed, the forces are again equal and opposite, i.e. the buoyancy force acting vertically upwards will be equal and opposite to the weight plus the air resistance acting vertically downwards.

Should, however, the upward speed of the balloon be increasing, the buoyancy force will be greater than the weight plus air resistance.

This increase in upward force is required to overcome the inertia of the body, and is equal to the product of its mass and acceleration.

Moments

A body may be acted on by two equal and opposite forces and yet not be in equilibrium; the points of application of the forces must be taken into account.

Suppose the body in Fig. 1 is hinged at one end, its weight will act vertically downwards from the C.G. and there will be a reaction at the hinge equal to the weight acting vertically upwards.

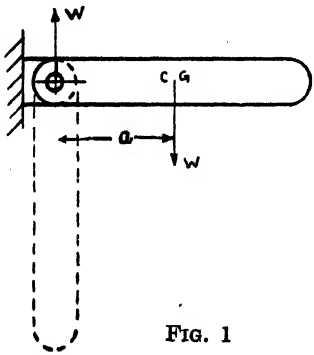


FIG. 1

It is obvious that the body will turn about the hinge until the centre of gravity is vertically under the hinge, i.e. until the forces are acting in the same straight line.

If, however, we apply a force R (Fig. 2) such that its tendency to turn the body in an anti-clockwise direction is equal to the tendency

of the weight of the body to turn it in a clockwise direction, equilibrium is maintained.

This tendency of force to turn a body about any point is called the moment of the force about that point, and is measured by the product of the force and the perpendicular distance of its line of action from the point.

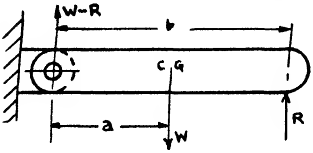


FIG. 2

It should be obvious that a force acting at a point has no moment about that point.

Thus, in Fig. 2 the turning effect of R about the hinge equals Rb and of W equals Wa. In order that the body may not turn, the two turning effects must be equal and opposite.

Thus $Rb - Wa = 0$.

I.e. the algebraic sum of the moments is zero.

Conditions of Equilibrium

A body will be in equilibrium, i.e. continue in its state of rest or motion—if—

(1) The algebraic sum of the forces equals zero in any direction.

(2) The algebraic sum of the moments equals zero about any point.

If the forces are greater in one direction than another, the body will accelerate in that direction.

If the moments about any point are greater in one direction than another, the body will accelerate round the point in that direction.

A single resultant force will not turn a body. If a body has a fulcrum, as in Fig. 1, the application of the force W produces an opposite force W at the fulcrum, and this gives an unbalanced moment or couple Wa, and the body accelerates round the fulcrum.

A body such as an aeroplane in the air, which is free to turn about any point, will turn about its centre of inertia or C.G. An unbalanced force then causes acceleration, and the inertia of the aeroplane's mass acting at the C.G. produces the couple depending on the distance from the C.G. and magnitude of the resultant force.

EXAMPLE 1. A boy weighing 50 lb. and his father weighing 150 lb. are on a see-saw. If the distance between the boy and his father is 12 ft., find how far the support is from the boy, also the reaction if equilibrium is maintained. Neglect the weight of the plank.

$$\begin{aligned} \text{Upward force} &= \text{Downward force.} \\ \text{Reaction} &= 50 + 150 \\ &= 200 \text{ lb.} \end{aligned}$$

Let x equal the distance of the support from the boy.

Taking moments about the boy's end—

$$12 \times 150 - 200\,x = 0$$

$$\therefore\ x = \frac{150 \times 12}{200} = 9 \text{ ft.}$$

$$\begin{aligned} \text{Reaction of support} &= 200 \text{ lb.} \\ \text{Distance of support from boy} &= 9 \text{ ft.} \end{aligned}$$

EXAMPLE 2. Two men are turning a capstan at a steady speed,

one man exerts a push of 80 lb. at a distance of 6 ft. from the centre and the other 100 lb. at 5 ft. They are winding a rope round the capstan at a radius of 6 in. Find the tension in the rope and the horizontal reaction of the capstan. Neglect friction.

Taking moments about the centre—

$$80 \times 6 + 100 \times 5 - T \times \tfrac{1}{2} = 0$$

$$480 + 500 = \frac{T}{2}$$

$$\text{Tension } T = 980 \times 2 = \underline{1960 \text{ lb.}}$$

$$R + 100 - T - 80 = 0$$

$$R = T + 80 - 100$$

$$R = 1960 + 80 - 100$$

$$\text{Reaction } R = \underline{1940 \text{ lb.}}$$

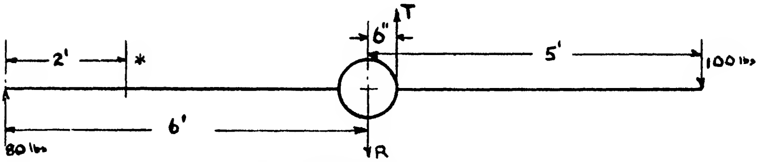


FIG. EXAMPLE 2. CHAP. 1

Moments may be taken about any point. Taking them about 2 ft. from the end—

$$80 \times 2 + 4R + 100 \times 9 - 4{\cdot}5T = 0$$

$$160 + 4(T + 80 - 100) + 900 - 4{\cdot}5T = 0$$

$$160 + 4T - 80 + 900 - 4{\cdot}5T = 0$$

$$0{\cdot}5T = 980$$

$$T = \underline{1960 \text{ lb.}} \text{ (as before).}$$

Resolution of Forces

A force on a body may be replaced by two or more other suitable forces and the effect remain the same. It is often convenient in working problems where there is an oblique force to represent this force by two others acting at right angles.

FIG. 3

The force R (Fig. 3) may be represented by the straight line *ob* drawn parallel to the direction of the force and representing its

magnitude at some suitable scale. R will have the same effect as L and D taken together. The projection of ob on the line oa parallel to the line of action of L will represent the magnitude of L to the same scale as ob. This is known as the component of R in the direction oa. In the same way ab represents the component of R in the direction ab, i.e. the magnitude of D.

This may be done graphically or it may be calculated as follows—

$$oa = ob \cos a$$
$$\therefore\ L = R \cos a.$$
$$ab = ob \sin a$$
$$\therefore\ D = R \sin a.$$

Let it be required to resolve R in directions L and D not at right angles (Fig. 4).

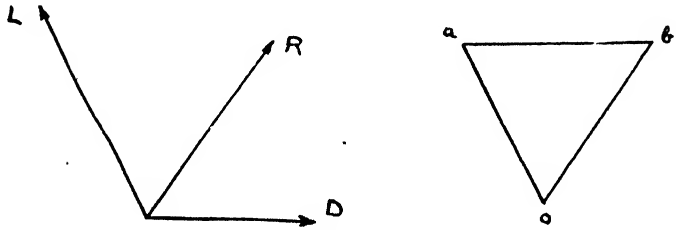


FIG. 4

Draw ob to represent R in direction and magnitude, and from o draw a line parallel to the direction of L and from b one parallel to the direction of D. Let them meet at a. Then oa and ab represent the forces L and D respectively.

As a corollary, L and D can be replaced by one force R. Draw oa representing L in direction and magnitude, and from a draw ab to represent D. Join ob. The line ob will give the direction and magnitude of R.

Polygon of Forces

If any number of forces in one plane acting at a point can be represented in direction and magnitude by the sides of a polygon taken in order, they are in equilibrium.

In Fig. 5 the four forces AB, BC, CD, and DA are represented in direction and magnitude by the closed polygon $ABCD$.

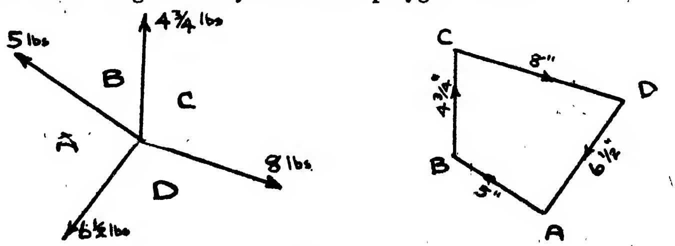


FIG. 5

In Fig. 6 are five forces not in equilibrium, as the polygon is not complete.

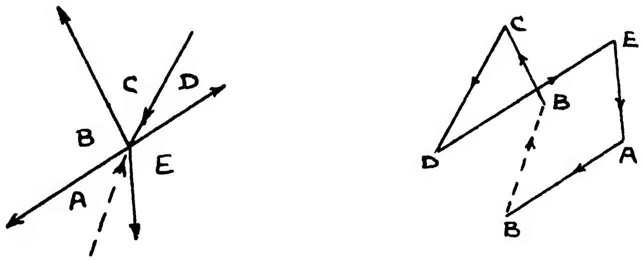


FIG. 6

The line *BB* which completes the polygon represents a force which, if added to the others, will put them in equilibrium. If the forces do not act at a point, the forces will be balanced if the polygon is closed, but the moments may not be.

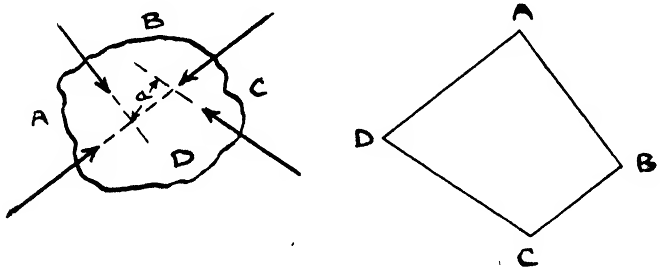


FIG. 7

Take, for example, the case in Fig. 7. The forces will be balanced, but taking moments about the point of intersection of lines of action of the forces *AB*, *BC*, and *DA*, it will be seen there is an unbalanced moment equal to *a CD*.

For three forces to be in equilibrium their lines of action must meet at a point.

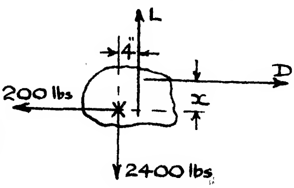


FIG. 8

EXAMPLE 1. If the body in Fig. 8 is in equilibrium, find L, D, and x.

Vertical and horizontal forces must balance, therefore—

$$D = 200 \text{ lb.}$$
$$L = 2400 \text{ lb.}$$

Taking moments about X

$$Dx - 4L = 0$$
$$200x - 4 \times 2400 = 0$$
$$x = \frac{9600}{200} = \underline{48 \text{ in.}}$$

EXAMPLE 2. If the body shown in Fig. 9 is in equilibrium, find T, D, W, and P.

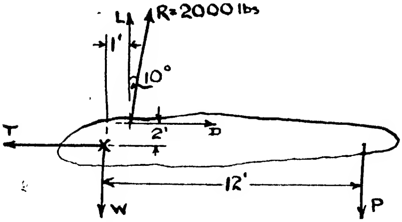


FIG. 9

Resolving R in the vertical and horizontal direction—

$$\begin{aligned}\text{Vertical component of } R = L &= R\cos a\\ &= 2000\cos 10°\\ &= 2000 \times 0{\cdot}9848\\ &= 1969{\cdot}6 \text{ lb.}\end{aligned}$$

$$\begin{aligned}\text{Horizontal component of } R &= D\\ &= 2000\sin 10°\\ &= 2000 \times 0{\cdot}1736\\ &= 347{\cdot}2 \text{ lb.}\end{aligned}$$

$$T = D = 347{\cdot}2 \text{ lb.}$$

Taking moments about X—

$$\begin{aligned}12P - L + 2D &= 0\\ 12P - 1969{\cdot}6 + 694{\cdot}4 &= 0\\ P &= \frac{1275{\cdot}2}{12}\\ &= 106{\cdot}3 \text{ lb.}\end{aligned}$$

$$\begin{aligned}W + P &= L\\ W &= 1969{\cdot}6 - 106{\cdot}3\\ &= 1863{\cdot}3 \text{ lb.}\end{aligned}$$

EXAMPLE 3. Fig. 10 is a diagrammatic sketch of a fitting, attached to which, on a radius of 2 in., is a tie A and two struts B and C. Another strut is to be fitted such that the fitting will be in equilibrium.

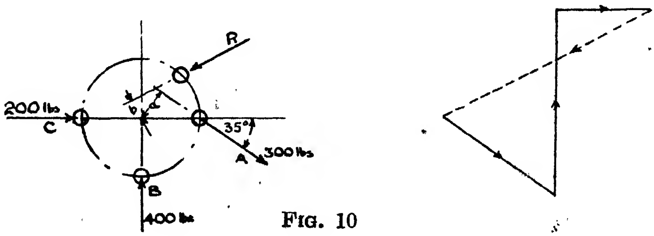


FIG. 10

Find the position, direction, and the force in this strut.

Draw the fitting to scale. Represent A, B, and C by an enclosed force polygon, then the line joining the enclosed ends will represent the unknown force R in direction and magnitude.

This gives the force $R = \underline{500 \text{ lb.}}$, and its direction $= \underline{27°}$ to the horizontal.

Taking moments about the centre,

$$Aa - Rb = 0.$$

The perpendicular distance of the line of action of A may be scaled, and is found to equal 1·15 in.

$$\text{Then } 300 \times 1{\cdot}15 - 500\,b = 0$$

$$b = \frac{345}{500} = \underline{0{\cdot}69 \text{ in.}}$$

This fixes the position of R.

Moment of Inertia

The couple required to make a mass accelerate round an axis is proportional to a quantity called the Moment of Inertia, which depends on the mass and its distribution with regard to the axis.

The couple is equal to the product of the angular acceleration and the moment of inertia.

The moment of inertia about any axis of a mass concentrated at a point is defined as the product of the mass and the square of its distance from the axis.

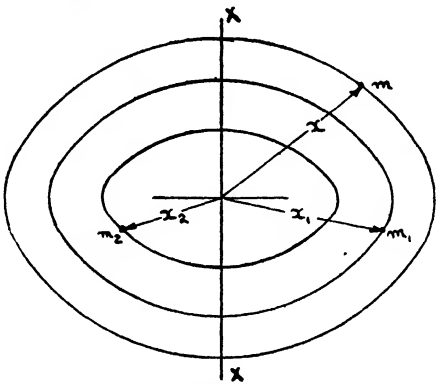


FIG. 11

In Fig. 11 the moment of inertia of the mass m about the axis xx is mx^2.

The moment of inertia of the mass m_1 about the same axis is $m_1x_1^2$, and of m_2, $m_2x_2^2$. The total moment of inertia of the three masses $= mx^2 + m_1x_1^2 + m_2x_2^2$.

It should be noted that the moment of inertia is not the product of the total mass and the square of the distance of the C.G., but the sum

of the products of all the small elements of mass and the square of their respective distances from the axis.

The moment of inertia about an axis xx is usually written $I\ xx$.

Consider the mass of material in Fig. 12 to be rotating round axis xx, then in order to find $I\ xx$ it must be considered split up into a number of infinitely small elements m, m_1, m_2, etc., and the moment of inertia of each of these found and the whole added together.

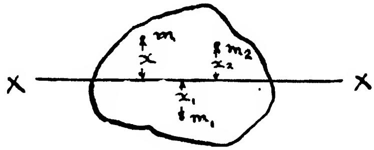


FIG. 12

Thus $I\ xx = mx^2 + m_1x_1^2 + m_2x_2^2 +$ etc.

This is usually written

$$I\ xx = \Sigma\ mx^2,$$

meaning the sum of the products of all the elements of mass making up the whole mass and the square of their respective distances.

This cannot be done by simple mathematics, but a very close approximation may be made by dividing the mass up into small instead of infinitely small elements.

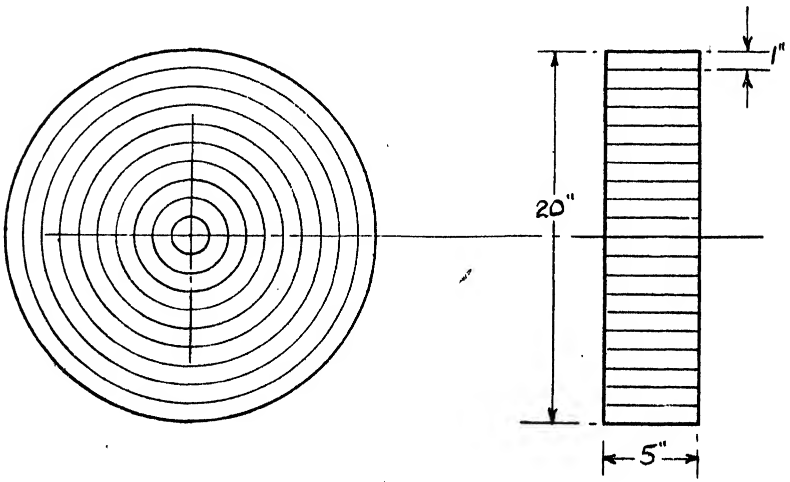


FIG. 13

Take, for example, a solid iron flywheel. Its moment of inertia may be found approximately by dividing it up into a number of cylindrical parts as shown in Fig. 13, finding I for each and adding them together.

Weight of iron = 0·26 lb./cu. in.

Number of Cylinder from outside	Mass $5\pi\,(R^2 - r^2)\,0{\cdot}26$	Distance² (from axis to centre of material)	$I = mx^2$
1	77·56	90·25	6997
2	69·39	72·25	5013
3	61·23	56·25	3444
4	53·07	42·25	2242
5	44·91	30·25	1359
6	36·75	20·25	744
7	28·59	12·25	350
8	20·41	6·25	128
9	12·25	2·25	28
10	4·08	0·25	1

Total I = 20306 lb. in.²

The correct solution is 20410 lb. in.², from which it will be seen that the above approximation is only 0·5 per cent out.

Radius of Gyration

By dividing I by the mass we get the square of the distance from the axis to the circumference of a circle at which, if all the materials of a rotating body could be concentrated, its effect would remain unchanged.

This is called the Radius of Gyration, and may be defined as the radius at which, if all the mass were concentrated, the action of a rotating body would be unaltered.

$$I = MK^2,$$

where M = the total mass and K = Radius of Gyration.

$K = \dfrac{r}{\sqrt{2}}$ for a solid cylinder and

$\sqrt{\dfrac{r^2 + r_1^2}{2}}$ for a hollow cylinder about the longitudinal axis,

where r = external radius

r_1 = internal radius.

Referring to the previous example of the flywheel—

$$\text{Total mass} = 5 \times 10^2 \times \pi \times 0{\cdot}26$$
$$= 408{\cdot}2 \text{ lb.}$$
$$I = MK^2$$
$$= \frac{Mr^2}{2}$$
$$\doteq \frac{408{\cdot}2 \times 100}{2}$$
$$= \underline{20410 \text{ lb. in.}^2}$$

Example. Find the moment of inertia of a hollow cylinder about *its longitudinal axis, given*—

Total weight of cylinder = 50 lb.
External diameter = 4 in.
Internal diameter = 3 in.
Height = 10 in.

$$I = MK^2$$
$$= M\left(\frac{r^2 + r_1^2}{2}\right)$$
$$= \frac{50 \times (16 + 9)}{2}$$
$$= 625 \text{ lb. in.}^2$$

Moment of Inertia of an Area

Although an area can have no inertia, the product of a small element of area and the square of its distance from an axis is called the Moment of Inertia or second moment of the element of area about that axis.

In the same way as previously explained for mass, the total moment of inertia of an area is equal to the sum of the products of all the elements of area making up the whole and the square of their respective distances from the axis considered.

Referring to Fig. 14—

$$I\,xx = ax^2 + a_1x_1^2 + a_2x_2^2 + \text{etc.}$$
$$= \Sigma\, ax^2.$$

In this case

$$I = AK^2,$$

where A is the total area.

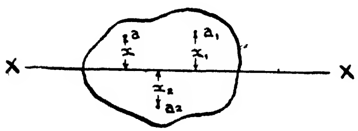


Fig. 14

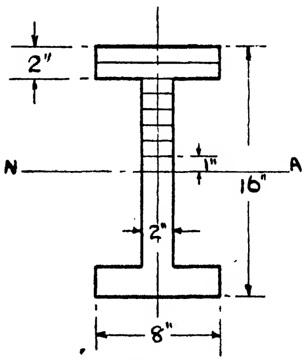


Fig. 15

The following example will make this clear.

Find the moment of inertia of the area about the axis through the centroid of the I section in Fig. 15, by the approximate method previously used for the mass moment of inertia.

Number of Section from outside	a Area of Section (in.^{2})	x^2 (Distance)^2 (from NA)	$I_{NA} = ax^2$
1	8	56·25	450
2	8	42·25	338
3	2	30·25	60·5
4	2	20·25	40·5
5	2	12·25	24·5
6	2	6·25	12·5
7	2	2·25	4·5
8	2	0·25	0·5

$$I \text{ for half section} = \underline{931 \text{ in.}^4}$$

$$\begin{aligned}\text{Total } I_{NA} &= 931 \times 2\\ &= \underline{1862 \text{ in.}^4}\end{aligned}$$

The Radius of Gyration will equal

$$\begin{aligned}&\sqrt{\frac{I}{A}}\\ &= \sqrt{\frac{1862}{8 \times 4 + 12 \times 2}}\\ &= \sqrt{\frac{1862}{56}}\\ &= \sqrt{33{\cdot}25}\\ &= \underline{5{\cdot}77 \text{ in.}}\end{aligned}$$

The moment of inertia of an area about an axis through the centroid is required in dealing with the strength of beams and struts.

Most engineering pocket-books give the moments of inertia for simple standard sections in terms of their dimensions.

Examples are given in Appendix I, also a method of finding I for a thin metal section in Appendix II.

It will be seen that for an I section

$$I = \frac{bd^3 - b_1d_1^3}{12}$$

Using this to solve the case in the previous example—

$$\begin{aligned}I &= \frac{8 \times 16^3 - 6 \times 12^3}{12}\\ &= \frac{32768 - 10368}{12}\\ &= \frac{22400}{12} = \underline{1867 \text{ in.}^4}\\ K &= \sqrt{\frac{1867}{56}}\\ &= \sqrt{33{\cdot}34}\\ &= \underline{5{\cdot}77 \text{ in.}}\end{aligned}$$

CHAPTER II

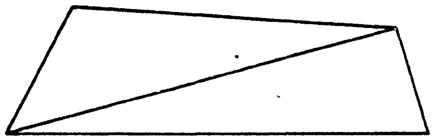
FRAMES

THE name Frame is given to a structure consisting of a number of struts and ties pin-jointed together carrying loads mainly at the joints. Its individual members are in compression or tension, although as a whole it may be subjected to bending.

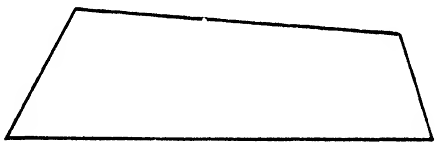
If the loads and members all lie in one plane, it is called a Plane Frame; and if in more than one plane, a Space Frame.

The most important frames in the aeroplane act as a whole as beams, and are named Trusses.

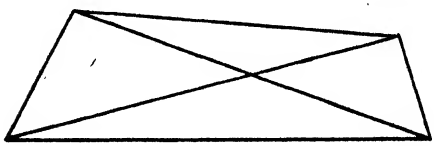
A perfect frame is one that has just enough members to keep it rigid under all systems of loading. If it has less it is said to be deficient, and if more it is called a Redundant Frame.



(a)



(b)



(c)

FIG. 16

Fig. 16 (*a*) shows a perfect frame, (*b*) a deficient frame, and (*c*) a redundant frame.

In Fig. 16 (*c*), if the cross members would only take a tensile load,

it would be a perfect frame if when one was under load the other was slack.

This type of bracing is considerably employed in aeroplane structures.

Where cross-bracing is employed care must be taken that one tie is not tensioned against the other or the frame will become self-strained, i.e. the members would have initial loads in them, which, with the addition of the loads due to external forces, are liable to cause failure.

The basis of the perfect frame is the triangle, i.e. a perfect frame is made up of one or more triangles.

If a frame is in equilibrium, two conditions must be satisfied—

1. The external forces (loads applied) must be in equilibrium.
2. The external forces plus the internal forces (loads in the members) at any joint must be in equilibrium.

Method of Determining the Loads in the Members of a Frame

In the simple frame shown in Fig. 17 there will be one external and two internal forces acting at each joint.

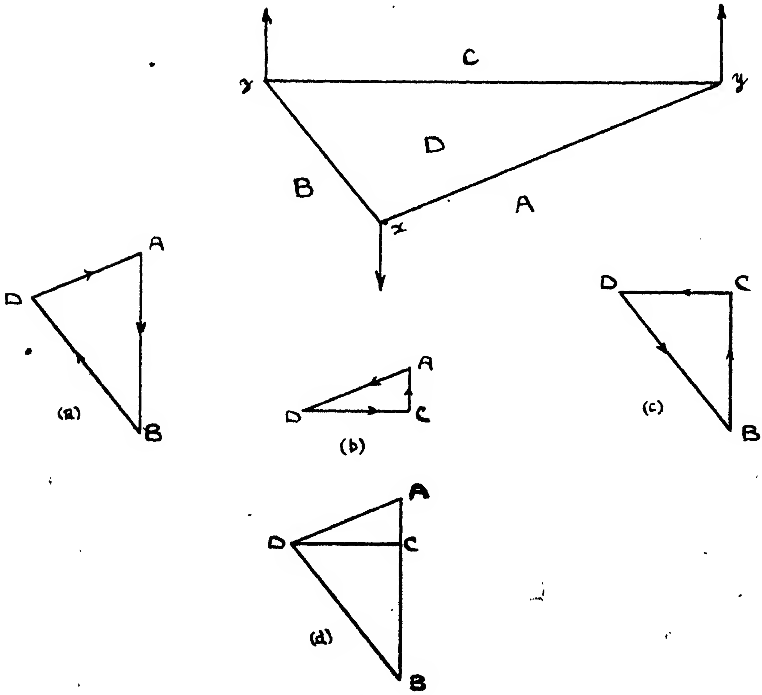


Fig. 17

If a letter is put between each external force and one in each triangle of the structure, the forces may be named by the letters each side of them, reading in a clockwise direction. This system of lettering was

devised by Henrici and Bow, and is known as Bow's Notation. Thus in the bottom joint x there is the external force AB and two internal forces BD and DA. These forces being in equilibrium, they may be represented by a closed triangle.

Set off a line AB (Fig. 17 (a)) vertically downwards to represent the force AB to a suitable scale. From B draw a line parallel to the internal force BD, and from A one parallel to the force DA; where these cross will be the point D. The length BD will represent the magnitude of the force BD, and DA the magnitude of the force DA.

The direction they are acting at the joint will be found by the direction of the lettering in the triangle. Thus the force BD is upwards as in the triangle B to D is upwards; in the same way, DA is upwards. This means there is a pull in these members away from the joint x, and they are therefore taking tension.

In like manner the internal forces at z and y may be found from the triangles (Fig. 17 (b) and (c)).

By putting these three triangles together, a single diagram is formed showing all the external forces and all the loads in the members.

This is called a Stress Diagram.

Stress Diagrams

By the use of Bow's Notation, a stress diagram may be drawn for a whole structure without considering separate force polygons.

An example will make the method clear.

Fig. 18 (a) is a line drawing, showing the external loading of the structure of which we wish to find the internal loads. It is very important that it should be drawn accurately to scale.

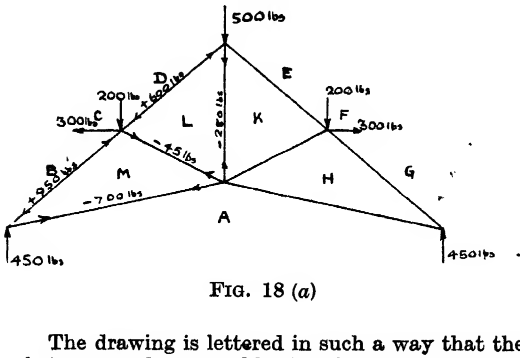


FIG. 18 (a)

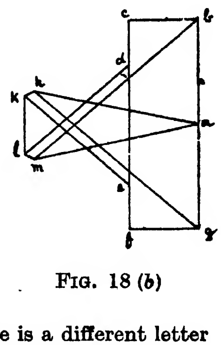


FIG. 18 (b)

The drawing is lettered in such a way that there is a different letter between each external load and one in each triangle of the structure.

The forces are known by the letters each side of them, reading in a clockwise direction, e.g. the left-hand 450 lb. load is known as AB, and not BA.

Note. It is not incorrect to read the loads anti-clockwise, but whichever method we choose must be used consistently throughout.

Commencing with the force AB, set down the line ab parallel to and in the direction of the force AB and representing to some suitable scale

its magnitude. Note the line goes from a to b upwards, as the force AB is upwards.

From b draw bc, parallel to and in the same direction as the force BC, and to the same scale as before.

This is continued for all the external forces in order, thus forming the figure a, b, c, d, e, f, g.

This will be a closed polygon, as the forces must be in equilibrium.

We now have to find the forces in the members, or internal forces.

We must start at a triangle of the structure that has two external members, so that we have two known points to draw vectors from and find where they cross. In this case we can start at triangles M or H but not L or K.

Starting at the triangle M, through a draw a line parallel to AM, and through b a line parallel to BM; where these meet will be the point m. Next through m draw a line parallel to ML, and through d a line parallel to DL; where they meet will be l. Continue this until k and h are found, when the stress diagram is complete (Fig. 18 (b)).

The forces in all the members may now be found by measuring the lengths of the corresponding lines in the stress diagram, e.g. the length of bm represents the force in BM to the same scale as that used for the external diagram.

Having found all the forces in this manner, we next get the direction they are acting at the joints. If they are pulling away from a joint, the member is in tension; and if pushing into the joint, compression.

Taking the extreme left-hand joint, we have the forces BM and MA. (*Note.* These are named in a clockwise direction.) Referring to the diagram, we see that b to m is downwards, therefore the force BM at that joint is downwards, i.e. into the joint or compression. ma is to the right, therefore the force MA is to the right or away from the joint, and the member is in tension. This is repeated for each joint and the direction of the force found in each case.

It is usual to distinguish the tensile from the compressive loads by calling the tensile "negative" and the compressive "positive."

Note. In this example capital letters are used to represent the forces in the structure and the corresponding small letters for the same forces in the stress diagram. This is simply for ease of reference; in examples no distinction will be made.

Some people prefer to use two types of letters. This practice will not, however, be adopted here, as it is not the system devised by Henrici and Bow; also it leads to complications on advanced work, where more than 26 letters are required.

Cross Bracing

Where cross bracing is used, the wire which is thought to be in compression must be omitted in drawing the stress diagram.

If the wrong wire is chosen, it will be found to show a compression, in which case that portion of the diagram must be drawn again, using the other wire.

Example. Draw the stress diagram for the cross-braced structure illustrated in Fig. 18 (c).

Consider the right-hand frame: the end strut tends to rise due to

the up load of $8\frac{1}{2}$ tons minus the down load of 2 tons. This will obviously be prevented by the cross bracing shown by a full line. Now if we consider the next frame, we see that the strut GH tends to go down due to the loads to the right having a resultant downwards. Thus the wire shown dotted will be in tension.

If the top or bottom member had been sloping we could not have found the correct wire so readily, as the force in the member would have had an upward or downward component. However, we can get

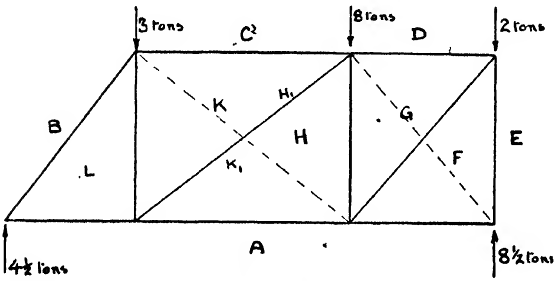


FIG. 18 (c)

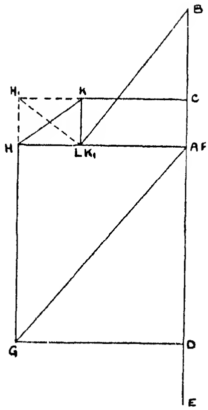


FIG. 18 (d)

a rough idea of the wire which is in tension by finding the direction of the resultant force on one side of the frame considered. Then if the wrong wire has been chosen, that part of the stress diagram should be redrawn, using the other wire.

In this example we will take the wrong wire HK (full line). The stress diagram is shown in Fig. 18 (d). This shows HK in compression, so the other wire H_1K_1 is now taken. The difference to the diagram is shown dotted.

Special Cases

If a force is applied other than at a joint, it may be replaced by two other forces acting at the joints. This must be done before drawing the stress diagram. Assuming a force of 30 lb. acts on a bar AB, 2 ft. from B and 1 ft. from A.

$$\text{Equivalent force at } A = \frac{30 \times 2}{3} = 20 \text{ lb.}$$

$$\text{,,} \quad \text{,,} \quad B = 30 - 20 = 10 \text{ lb.}$$

These forces have the same effect on the other members of the structure as the 30 lb. The 30 lb. will, however, put a bending moment on the bar AB, which must be taken into account when designing AB.

External forces must be drawn outside the frame, and P in Fig. 18 (*e*) should be drawn outside as P_1.

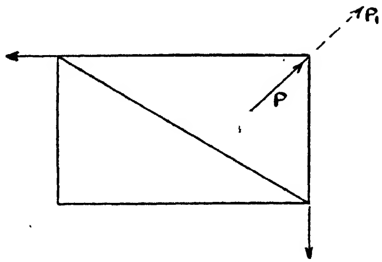


FIG. 18 (*e*)

Incomplete Diagrams

It is often convenient when dealing with part of a structure to draw an incomplete diagram, particularly in the case of structures supported at one end.

Assume we wish to find the loads in the flying wires and spars of the biplane truss shown in Fig. 19 (*a*).

The complete diagram is shown in Fig. 19 (*b*), but all that is necessary

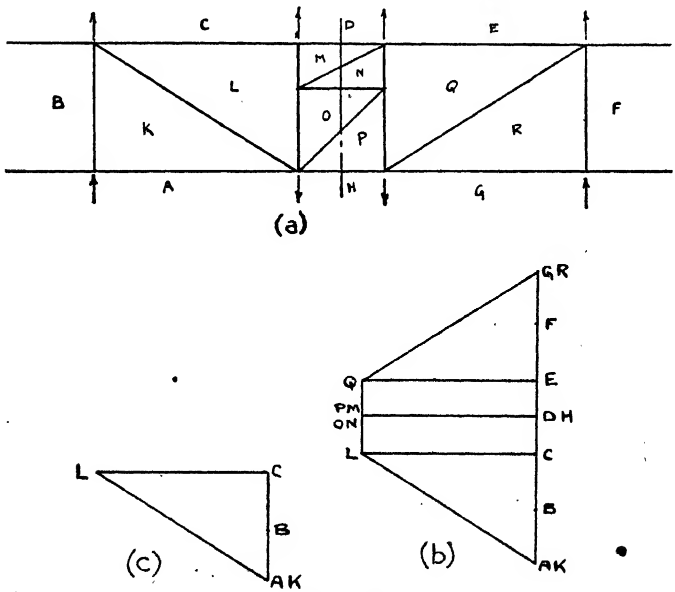


FIG. 19

in order to obtain the loads required may be found from the portion of the diagram Fig. 19 (*c*). Fig. 22 (*c*) is another example of this type of diagram.

Method of Sections

This method enables the load in any member of a structure to be found without having to first find the loads in several other members. It is useful also as a check upon the accuracy of results obtained from a stress diagram.

It becomes rather laborious when there are many external loads at different angles, and is therefore not recommended for use when loads in several members are required.

The method is a development of the fact that a structure in equilibrium which is divided into two by an ideal section has its external forces on one side of the section, balanced by the forces acting in the members cut by the section.

This will be best illustrated by an example.

Take for example the tail end of a fuselage having only one external load P at that end (Fig. 20 (*a*)). Assume the third bay to be cut in two by the line xx, then in order that the structure may be in equilibrium the algebraic sum of the moments of the forces in the members AB, AC, and DC, together with the external force P, must equal zero about any point. This is clear if we consider the two rear bays as a solid body (Fig. 20 (*b*)). It is in equilibrium under the forces P, AB, AC, and $C.D$

(*Note.* Bow's Notation is not convenient here.)

To find the force in AB, take moments about C, thus eliminating AC and DC, which have no moment about this point. Assume the force in AB acts at the point B in the direction given by the arrow.

Then $xP - a \times AB = 0$

or $$AB = \frac{xP}{a}$$

If this answer should be negative, the force in AB will be in the opposite direction to that shown by the arrow.

To find DC take moments about A,

then $yP - c \times DC = 0$

or $$DC = \frac{yP}{c}$$

To find AC take moments about D,

then $yP - b \times AB - d \times AC = 0$

or $$AC = \frac{yP - b \times AB}{d}$$

$$= \frac{yP - b \times \frac{xP}{a}}{d}$$

Note. AB must be found before AC.

To find BC assume the structure is cut by the line $x_1 - x_1$, then taking moments about E

$$zP + e \times BC - f \times AB = 0$$

or

$$BC = \frac{f \times AB - zP}{e}$$

$$= \frac{f\dfrac{xP}{a} - zP}{e}$$

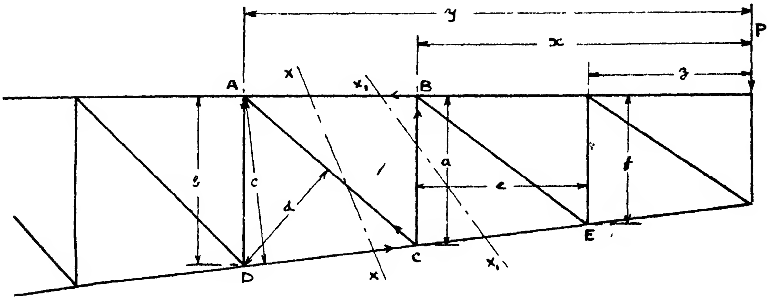


FIG. 20 (a)

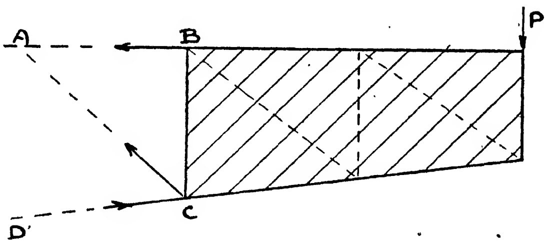


FIG. 20 (b)

In this way, proceeding bay by bay, the forces in all the members may be found.

EXAMPLE 1. Find by the method of sections the loads in the members BC, BE, and DE of the biplane truss shown in Fig. 21.

Assume the truss is cut in two by a line cutting BC, BE and DE, and DE to be in tension as shown by the arrow.

Taking moments about B,

$$5DE - 4{\cdot}5 \times 120 - 8{\cdot}5 \times 100 = 0$$

$$DE = \frac{540 \times 850}{5}$$

$$= 278 \text{ lb.}$$

As this is positive, the load in DE = 278 lb. tension.

BC is obviously in compression, but as an example we will assume it to be in tension or pulling from B.

Taking moments about E,

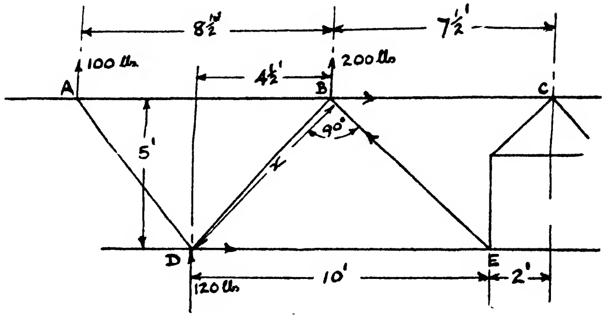


FIG. 21

$$5BC + 10 \times 120 + 5\tfrac{1}{2} \times 200 + 14 \times 100 = 0$$

$$BC = \frac{-1200 - 1100 - 1400}{5}$$
$$= -740 \text{ lb.}$$

The negative answer shows that the assumption that BC was tension was wrong, and the load in BC = 740 lb. compression.

Assume BE is in tension.

Taking moments about D,

$$xBE - 5BC - 4{\cdot}5 \times 200 + 4 \times 100 = 0$$

x, the distance of the line of action of BE from D, may be scaled from an accurate drawing and found to equal $6\tfrac{3}{4}$ ft.

Then
$$6{\cdot}75BE - 5BC - 4{\cdot}5 \times 200 + 4 \times 100 = 0$$
$$6{\cdot}75BE - 5 \times 740 - 900 + 400 = 0$$
$$BE = \frac{3700 + 900 - 400}{6{\cdot}75}$$
$$= \frac{4200}{6{\cdot}75}$$
$$= 622 \text{ lb.}$$

i.e. BE has a load of 622 lb. tension.

EXAMPLE 2. Find the loads in all the members of the tail-end of the fuselage shown in Fig. 22 (a), by the stress diagram method.

The stress diagram is shown in (b), and the loads obtained from it are shown against their respective members; negative showing tension.

EXAMPLE 3. Find the load in the member YX of the front portion of the rib shown diagrammatically in Fig. 22 (c).

By method of sections.

Let x = compressive load in top inner boom.

Perpendicular distance of top inner boom from Y = 5·1 in.

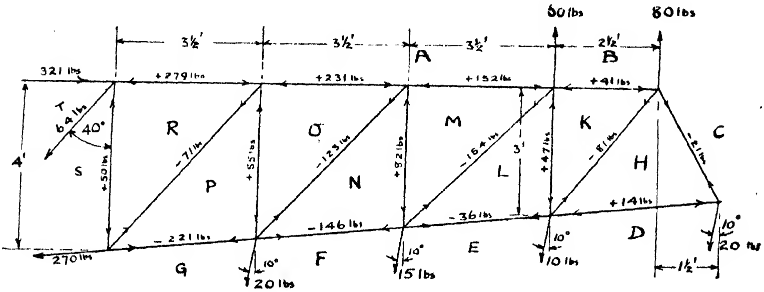


FIG. 22 (a)

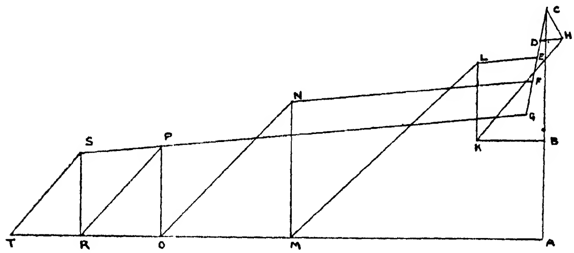


FIG. 22 (b)

Taking moments about Y—

$$48 \times 6{\cdot}5 + 77 \times 9 + 122 \times 1 - 5{\cdot}1x = 0$$

$$x = \frac{312 + 693 + 122}{5{\cdot}1}$$

$$= \frac{1127}{5{\cdot}1}$$

$$= \underline{221 \text{ lb.}}$$

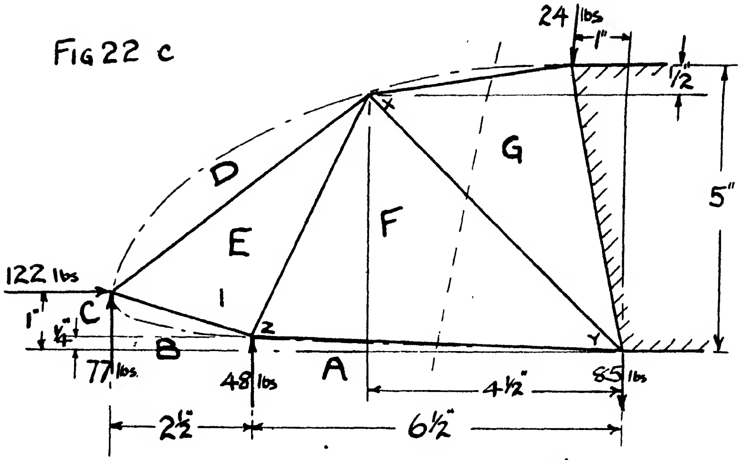
Perpendicular distance of top inner boom from Z = 3·93 in.

" " of YX from Z = 4·43 in.

Taking moments about Z—

$$77 \times 2{\cdot}5 + 122 \times 0{\cdot}75 - 3{\cdot}93 \times 221 + 4{\cdot}43 YX = 0$$

FIG 22 c



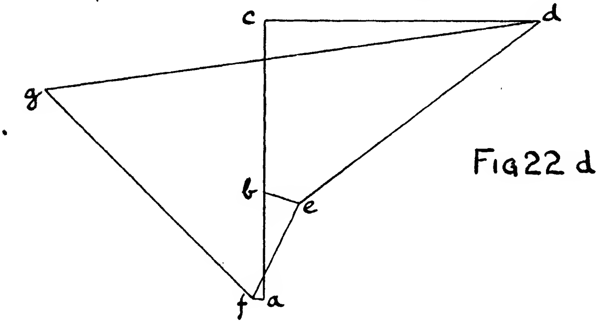


FIG 22 d

$$\text{Tensile load in } YX = \frac{192{\cdot}5 + 91{\cdot}5 - 868{\cdot}5}{4{\cdot}43}$$

$$= \frac{584{\cdot}5}{4{\cdot}43}$$

$$= \underline{132 \text{ lb.}}$$

The stress diagram is shown in Fig. 22 (*d*), for those who prefer that method.

It should be noted that when finding the loads in curved members, such as the rib booms, the loads are taken to act as though the members were on a straight line from joint to joint.

CHAPTER III

THE GENERAL PROBLEM

Weight of Aircraft in Relation to Structure.

THE factor of primary importance in aeroplane structural design is the necessity of obtaining maximum strength with a minimum of weight. With a reduction in structure weight, other things being equal, any of the following may be obtained—

1. Greater useful load.
2. Higher speed.
3. Higher rate of climb and ceiling.
4. Greater range.

An engineer is often said to be a man who can make for one shilling what anyone else could make for one pound sterling. An aeronautical engineer must be able to build with one ounce what any other engineer could build with one pound weight.

In dealing with the strength/weight ratio of aircraft it must be borne in mind that a reduction of weight at the expense of head resistance is not necessarily a gain in overall efficiency, especially with the modern high-speed aircraft.

Take, for example, a machine weighing 6000 lb. and having a L/D under certain conditions of 10, the thrust required would be 600 lb. for steady horizontal flight.

If by certain alterations the weight is reduced to 5850 lb., but at the same time the drag is increased so that the L/D is now 9, the thrust required to overcome this increased resistance would be 650 lb., i.e. more power would be required from the engine for the same performance.

Relative Weights

The total weight of a machine may be divided into four parts—

(*a*) Structural.
(*b*) Power plant.
(*c*) Useful load.
(*d*) Consumable load (fuel, oil, water, etc.).

The structure weight is usually about 35 per cent of the total. This may be subdivided into—

Wings	16 per cent	approx.
Body	12 ,,	
Tail unit	$2\frac{1}{2}$,,	
Chassis	$4\frac{1}{2}$,,	

The relative proportions of *b*, *c*, and *d* depend on the purpose for which the machine is to be used.

A commercial machine will have a high useful load and a relatively small power plant. If high speed is required, then the power plant weight will be high at the expense of useful load. For a long-range machine the consumable load will be high and the useful load small in proportion.

The Power Plant

Although the aircraft designers must rely on the automobile engineer to supply him with this important unit, he will, by choosing the best engine for the purpose for which the machine is to be used, considerably increase the performance.

Steady development has been and is being made by the engine constructor. Not only are engines more reliable, but their weight per horse-power has undergone definite decreases. In 1915 an average value for engine weight was 4 lb./b.h.p.; in 1920 this was reduced to 1·8 lb./b.h.p.; while at present it is only 1·5 lb./b.h.p.

This has been brought about largely by metallurgical progress, which has permitted the use of super-grade steels and light alloys, increase in thermal efficiency due to the use of higher compression ratios, made possible by the addition of "dopes" in the fuel, delaying detonation, use of variable pitch airscrews, supercharging, and gearing.

Consumable Load

Reduction in this item will depend on the use of the most efficient power plant and fuel, and a high lift/drag.

Structure Weight

It is in the structure that the designer must explore to the full all means by which the weight and head resistance of the machine may be reduced so as to enable greater speeds, increased ranges, more useful load, and higher altitudes to be obtained.

It will be interesting to note here how performance has been improved by reference to the following record figures—

	Speed m.p.h.	Non-stop Flight. Miles
1907	—	½
1913	126	634
1920	194	1,190
1925	280	3,200
1929	357·7	4,912
1933	423·7	5,657
1937	440·7	6,305·7
1939	463	7,162

Most important is the recent increase in speed of general types of aircraft. As horse-power is proportional to velocity cubed, it becomes vitally important to have a structure of good streamline shape and smooth surface in order to save horse-power by cutting down resistance to a minimum. In most cases reduction of head resistance means increased weight, but the higher the speed the greater the reduction of drag for a given increase in weight. For example at 150 m.p.h. the increase in weight involved by retracting the undercarriage prevents an improvement of performance, but at 300 m.p.h. four times the drag is saved ($D \propto V^2$) and it becomes well worth while. It is interesting to note that the Bristol Company designed a racing aeroplane with retractable undercarriage in 1922, so it is not a new idea. A high degree

of streamlining is only worth while on high-speed aircraft, and it is for this reason that only racing aeroplanes were highly streamlined in the past. Now that all aeroplanes have a high speed they are all well streamlined.

The chief advance has been in the development of the stressed-skin monoplane, which gives a smooth surface and enables a good streamline shape to be obtained without the use of fairings. At the same time this type of structure provides room in the wings for retraction of undercarriage and stowage of bombs, which until recently were carried in the open. It also requires no external bracings.

It was not until higher speeds made streamlining so important, and the consequent increase of wing loadings allowed a sturdier structure to be used without increase in the proportion of structure weight, that stressed skin was an advantage. In a lightly loaded slow machine, metal-stressed skin is likely to work out heavier than a braced structure, and at slow speeds weight has proportionally more importance than reduction of head resistance. Also higher speeds increase the tendency to flutter, and the stiffer stressed skin and geodetic structures decrease this tendency.

Materials

The majority of modern types of aircraft are made of aluminium alloy, steel being too heavy for stressed skin. Magnesium alloy, which would give a lighter structure than aluminium alloy, would be used if its low resistance to corrosion could be overcome. There are a few machines made of wood or partly of wood, and there are signs of a partial return to wood construction. Wood lends itself very well to stressed-skin construction, and the latest reinforced woods are very reliable.

A lot of experimental work is being done on plastic materials, and a few have given reasonable results, so that it may not be very long before these materials are used in the structure.

CHAPTER IV

LOADS ON AN AEROPLANE

When an aeroplane is flying level at constant speed, the supporting forces on the wings and other lifting surfaces will be exactly equal to the weight of the aeroplane, and the thrust will equal the total drag.

Suppose an aeroplane keeping its flying attitude suddenly has its thrust increased by opening up the engine. The result is an increase in forward speed with an acceleration upwards.

This acceleration throws extra forces on the wings.

All manoeuvres involve accelerations or decelerations, and these throw increased loads upon the structure.

Method of Obtaining Loads in the Structure

So intricate and variable are the external loads, arising as they do from air forces and the weight of the aeroplane itself, that to determine accurately all possible loads is a task that could only be attempted as a piece of scientific research.

For practical design purposes it is standard British practice to find the loads in the structure under certain conditions of steady flight and landing, e.g.—

1. Centre of Pressure Forward (minimum speed).
2. Centre of Pressure Back (maximum speed).
3. Nose Dive (terminal).
4. Landing.
5. Turning.
6. Inverted Flight (if necessary), etc.

These loads, called basic loads, are multiplied by a load factor which varies in each case and with the type of aeroplane.

Factor of Safety and Load Factor

In general engineering when designing a structure, the worst possible loads that can come on it are estimated, and these are multiplied by a figure called the Factor of Safety.

Thus the material under the worst conditions will only be stressed up to a fraction of its ultimate stress.

It is impracticable in aeronautical engineering to estimate the loads under all the conditions of flight. The loads which come on the machine under the above conditions of steady flight and landing are found, and these are multiplied by a figure called a Load Factor, and the member designed to withstand the resultant load.

There are two types of load factor, viz.—

1. *Ultimate Factor*. This is intended to allow for twice the loads expected during manoeuvres appropriate to the type of aeroplane. The structure must be designed so that it will not collapse before withstanding the product of the steady flight load and the ultimate factor.

2. *Proof Factor*. This is 75 per cent of the ultimate factor and is to allow for the structure being airworthy, i.e. without appreciable per-

manent set, under 1·5 times the maximum loads expected under manoeuvres appropriate to the type.

Ultimate factor

$$= \frac{2 \text{ maximum estimated load (load for which aeroplane is designed on ultimate stress)}}{\text{basic load}}$$

Proof factor

$$= \frac{1\tfrac{1}{2} \text{ maximum estimated load (load for which aeroplane is designed on proof or yield stress)}}{\text{basic load}}$$

It follows that if under any conditions an aeroplane is expected to carry x times its basic load in any of the cases given above, the structure must be given an ultimate factor of $2x$ for that particular case. The extra factor of 2 is to allow for emergency manoeuvres, faulty rigging, etc., and is not a factor of safety as in the general sense of the term, as loads are not always proportional to stress. This ensures that the aeroplane will be safe, though in many cases it will be stronger than is necessary. The human factor is the obstacle to an exact solution. The maximum loads encountered depend largely on how the pilot handles his craft. It is safe to say that a very clumsy pilot could break any aeroplane.

The designer does not himself estimate the load factor required. This is laid down by the Air Ministry. It is based on acceleration tests. Thus, if in an appropriate manoeuvre an aeroplane has an acceleration upwards of Mg, where g is the acceleration due to gravity, the upward force to produce this acceleration is $\frac{W}{g} . Mg = WM$ $\left(\text{force} = \frac{W}{g}a\right)$

In steady horizontal flight, Lift = Weight.

$$\frac{\text{Load in manoeuvre}}{\text{Load in steady flight}} = \frac{\text{Lift} + WM}{\text{Lift}} = \frac{W + WM}{W}$$

$$= 1 + M$$

Thus Ultimate factor $= (1 + M)2$

and Proof factor $= (1 + M)1{\cdot}5$

Example. A tensile member has to carry in steady horizontal flight a load of 5000 lb. in the C.P. Back case for which the ultimate factor is 5, and 4000 lb. in the C.P. Forward case for which the ultimate factor is 7. Find the minimum cross-sectional area of the member; given ultimate stress 52 tons/sq. in., proof stress 36 tons/sq. in.

Load corresponding to ultimate factor

1st case $= 5000 \times 5 = 25000$ lb.
2nd case $= 4000 \times 7 = 28000$ lb.

The second case is greater and so determines the maximum load the member must withstand. Load corresponding to proof factor in

2nd case $= 28000 \times 0{\cdot}75$
$= 21000$ lb.

The member must not exceed the proof or yield stress under this load

$$f = P/A$$

$$A = P/f = \frac{21000}{36 \times 2240}$$

$$= 0{\cdot}26 \text{ sq. in.}$$

For the load corresponding to the ultimate factor, use ultimate stress

$$A = P/f = \frac{28000}{52 \times 2240}$$
$$= 0{\cdot}24 \text{ sq. in.}$$

The larger will be the area required, i.e. 0·26 sq. in.

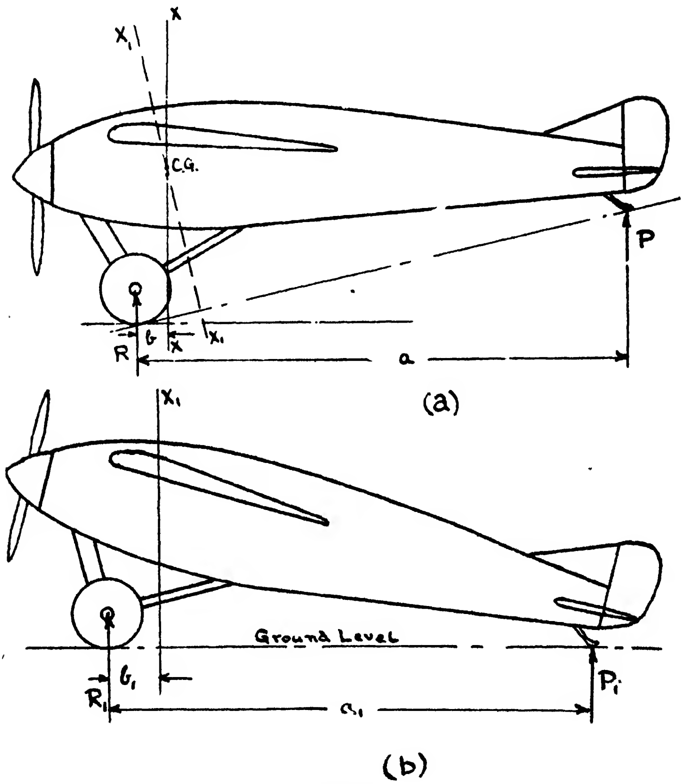


FIG. 24

Effect of Load Factor on Weight

A designer must make his machine to the required strength, but it is bad design to make it (apart from a small margin) any stronger. Extra strength means increased weight and decreased performance.

Major Barlow* has shown that a change of one unit in load factor, for any case, means approximately 1 per cent change in the structural weight, i.e. if a machine weighing 5000 lb. had its load factor changed

* *R.Ae.Soc. Journal*, April, 1929.

in the C.P. Forward case from 6 to 7, the resulting structure would weigh another 50 lb.

It follows that a machine required for aerobatics will be proportionally heavier than one required for steady flight.

A member made under usual workshop practice is called a *typical component.* Such a member is required to withstand on strength test 120 per cent of the factored load. A typical component is likely to be

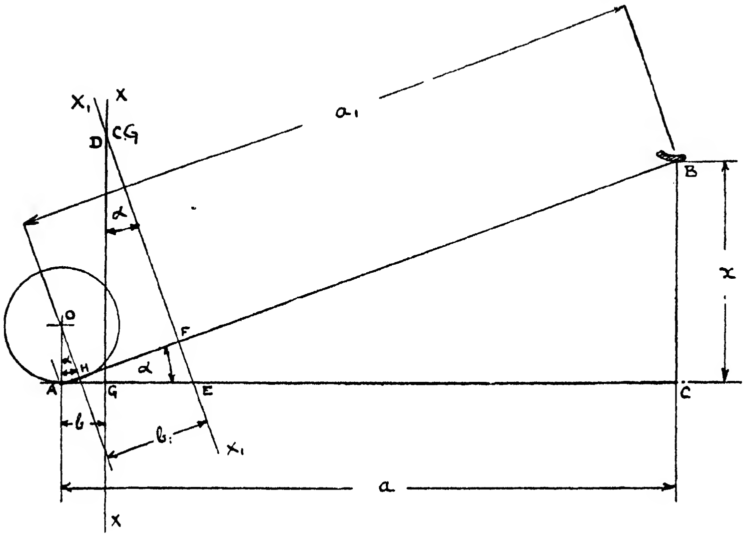


Fig. 25

stronger than a *standard component,* which is the weakest component complying with drawings, limits, and material specifications. If a typical component was tested and only just withstood the factored load, weaker components, still up to standard, would fail.

Centre of Gravity (Estimated)

In order to design a structure of suitable strength, and to get the Centre of Pressure in the right position, the weight of the aeroplane and position of the Centre of Gravity must be estimated.

This may only be found by a designer from experience of other machines.

As it is necessary to obtain this information before the shape or size of any detail is considered, there is often a considerable error in the result.

Where possible the thrust line should pass approximately through the C.G., in order to minimize the difference in tail load required to overcome the thrust moment due to engine on and off conditions.

If the C.G. is too far forward, the machine will be tail light and in landing tend to fall on its nose; if too far aft, it will be tail heavy,

which, apart from the flying qualities, will put increased loads on the fuselage in landing, due to extra weight being taken by the *tail skid.*

The relative positions of the C.G. and C.P. may be corrected by—

(*a*) Altering the fore and aft position of the wings or stagger.
(*b*) Sweep-back.
(*c*) Angle of the tail plane.
(*d*) Changing the position of the loads.

The latter method is not often possible owing to the great distance a load must be moved to appreciably affect the C.G.

(*a*), (*b*), and (*c*) will only correct the position of the C.G. with regard to the centre of pressure in flight.

If the error is great it will be necessary also to alter the position of the chassis so that the balance of the machine and strength of the fuselage will be correct in landing.

If the weight comes out less than was estimated, the performance will be improved, and *vice versa.*

Centre of Gravity (Actual)

In the finished machine it is found in the following manner.

The weight reactions on the wheels and skid are found in rigging position and again with the tail down (Fig. 24), weighbridges being placed under the wheels and a spring balance or weighbridge at the skid.

The distance behind the wheels of the vertical line XX through the C.G. in (*a*) is found by taking moments about R.

$$aP - b\,(R + P) = 0$$

or

$$b = \frac{aP}{W}$$

where W = total weight of machine.

In the same way the distance of the vertical line X_1X_1 behind the C.G. in (*b*)

$$b_1 = \frac{a_1P_1}{W}$$

Where XX and X_1X_1 cross gives the position of the C.G., and the height may be found by drawing Fig. 24 (*a*) to scale as in Fig. 25. Note that HB is the ground line in tail down position, and $X_1 - X_1$ is perpendicular to this line. b_1 is the distance HF and not AF, as the wheel reaction is along HO in tail down position.

The height may also be calculated from either of the following equations—

Referring to Fig. 25,

Let x = height of skid above ground in rigging position,
and r = radius of landing wheels (allowing for tyre deflection).

The other dimensions are as in Fig. 25.

Equation 1. ABC, DEG, OAH, and AEF are similar triangles.

$$\frac{DG}{GE} = \frac{a}{x}$$

$$DG = \frac{GE \times a}{x}$$

$$= \frac{(AE - b)a}{x}$$

$$DG = \frac{\left(\frac{AF(a_1 + AH)}{a} - b\right) a}{x}$$

$$= \frac{\left(\frac{(b_1 + AH)(a_1 + AH)}{a} - b\right) a}{x}$$

$$= \frac{\left(\frac{\left(b_1 + \frac{rx}{a}\right)\left(a_1 + \frac{rx}{a}\right)}{a} - b\right) a}{x}$$

$$= \frac{\left(\frac{a_1 P_1}{W} + \frac{rx}{a}\right)\left(a_1 + \frac{rx}{a}\right) - ab}{x}$$

= Height of C.G. above ground in rigging position.

Equation 2. In this case it is assumed that the point of support at the skid is the same in both positions.

$$b_1 = (DG - r) \sin a + b \cos a$$

$$a_1 = a \cos a + (x - r) \sin a$$

Taking moments about H

$$Wb_1 = P_1 a_1$$

$$W\,[(DG - r) \sin a + b \cos a] = P_1\,[a \cos a + (x - r) \sin a]$$

$$(DG - r) \sin a = \frac{P_1}{W}[a \cos a + (x - r) \sin a] - b \cos a$$

$$DG = \frac{\frac{P_1}{W}[a \cos a + (x - r) \sin a] - \frac{aP}{W} \cos a}{\sin a} + r.$$

$$= \frac{P_1 a}{W} \cot a + \frac{P_1(x - r)}{W} - \frac{aP \cot a}{W} + r$$

$$= \frac{a \cot a}{W}(P_1 - P) + \frac{P_1(x - r)}{W} + r$$

= Height of C.G. above ground in rigging position.

Note. Position should be given relative to axle in a defined position or to leading edge, as height above ground will change with load on wheels.

Equivalent Plane

In a biplane the forces upon the planes must be taken together to find the resultant action of the air on the wings.

It is usual to state the results in terms of an imaginary "equivalent plane," the position of which is such that the machine can be dealt with as a monoplane having all the lift concentrated in this one plane.

In Fig. 26

$$\frac{a}{b} = \frac{A_2 C_{L2}}{A_1 C_{L1}} = \frac{\text{Lift of bottom plane}}{\text{Lift of top plane}}$$

where A_1 and A_2 equal the areas of the top and bottom planes respectively and C_{L1} and C_{L2} their respective lift coefficients.

If the aerofoils are similar, C_{L1}/C_{L2} may be taken as 1·2.

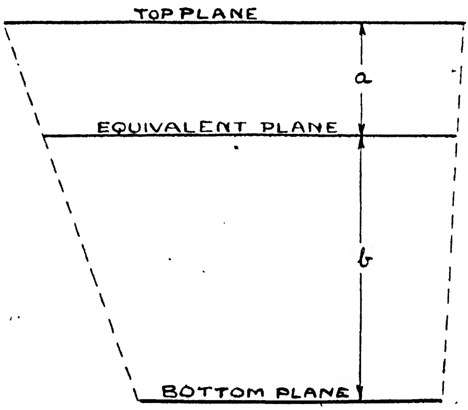


FIG. 26

EXAMPLE 1. The aeroplane in Fig. 27 is found to have its C.G. too far forward with regard to the main planes.

In order to correct this it is decided to move the top plane forward.

Find the correct distance of the leading edge of the top plane behind the centre line of the airscrew so that the C.G. will be 0·3 of the chord of the equivalent plane back from the leading edge.

Given: Movement of the C.G. forward due to correct movement of top wing = 1 in.

Aerofoils are similar and their areas are the same.

Distance of leading edge of equivalent plane in front of C.G. = 5·25 × 0·3 = 1·575 ft.

Distance of leading edge of bottom plane in front of C.G. = 11 in. = 0·917 ft.

Distance of leading edge of top plane in front of C.G.

$$= \frac{2 \cdot 2}{1 \cdot 2}(1 \cdot 575 - 0 \cdot 917) + 0 \cdot 917$$

$$= \underline{2 \cdot 123 \text{ ft., say 2 ft. } 1\tfrac{1}{2} \text{ in.}}$$

Distance of leading edge of top plane behind the centre line of airscrew

$$= 4 \text{ ft. } 11 \text{ in.} - 2 \text{ ft. } 1\tfrac{1}{2} \text{ in.}$$

$$= \underline{2 \text{ ft. } 9\tfrac{1}{2} \text{ in.}}$$

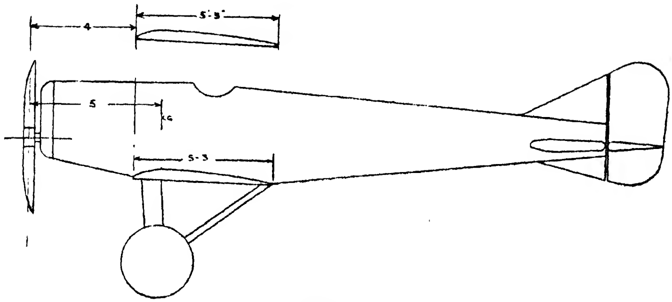


FIG. 27

EXAMPLE 2. An aeroplane when weighed in rigging position and again with the tail skid at ground level gave the following results—

Reaction on wheels in rigging position .	5800 lb.
Reaction on skid in rigging position .	200 lb.
Reaction on wheels, tail down position .	5600 lb.
Reaction on skid, tail down position .	400 lb.
Horizontal distance from wheels to skid in rigging position	30 ft.
Horizontal distance from wheels to skid with tail down	30·16 ft.
Height of skid above ground in rigging position	5 ft.
Diameter of landing wheels . . .	3 ft.

Find the height and horizontal distance of the C.G. behind the axle in rigging position.

Total weight $= 5800 + 200 = \underline{6000 \text{ lb.}}$

Distance of C.G. behind axle $= \dfrac{aP}{W}$

$$= \frac{30 \times 200}{6000} = \underline{1 \text{ ft.}}$$

$$\text{Height of C.G. above ground} = \frac{\left(\frac{a_1 P_1}{W} + \frac{rx}{a}\right)\left(a_1 + \frac{rx}{a}\right) - ab}{x}$$

$$= \frac{\left(\frac{30\cdot16 \times 400}{6000} + \frac{1\cdot5 \times 5}{30}\right)\left(30\cdot16 + \frac{1\cdot5 \times 5}{30}\right) - 30 \times 1}{5}$$

$$= \frac{(2\cdot01 + 0\cdot25)(30\cdot16 + 0\cdot25) - 30}{5}$$

$$= \frac{68\cdot73 - 30}{5}$$

$$= \underline{7\cdot75 \text{ ft.}}$$

Height above axle $= 7\cdot75 - 1\cdot5 = \underline{6\cdot25 \text{ ft.}}$

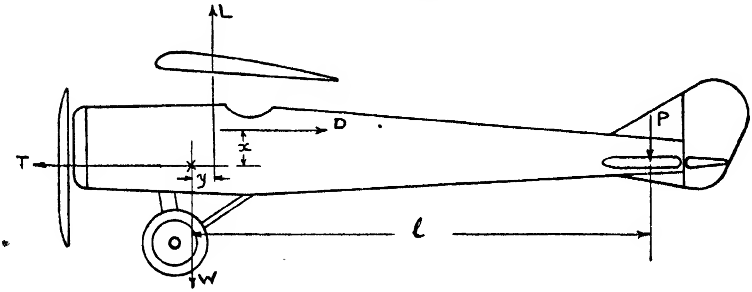


FIG. 28

Loads in Steady Horizontal Flight

Under this condition

Total lift = weight

Thrust of airscrew = total drag.

The sum of the moments about any point must be zero.

Referring to Fig. 28,

$$L - P = W$$
$$T = D$$

Taking moments about X,

$$Dx + Pl - Ly = 0$$

From these equations P and L may be found.

The tail load P may be either up or down, depending on the position of the centre of pressure. In high-speed flight the C.P. is back and there is a down load; while in low-speed flight the C.P. is forward, giving an up load on the tail.

EXAMPLE. An aeroplane weighing 12,000 lb. has its centre of pressure 8 in. behind the C.G., and the line of drag 2 ft. 8 in. above the thrust line, when in horizontal flight. Find the tail load, if the thrust is 1800 lb. and the C.P. of the tail plane is 42 ft. 8 in. behind the C.G.

Drag = thrust = 1800 lb.

Take moments about the point of intersection of the thrust and lift lines, and assume the tail load is upwards.

$$1800 \times 2\tfrac{2}{3} - 12000 \times \tfrac{2}{3} - 42P = 0$$

$$126P = 14400 - 24000$$

$$P = -\frac{9600}{126}$$

$$= 76{\cdot}2 \text{ lb. downwards.}$$

Nose Dive

The limiting case of a nose dive occurs when the aeroplane travels in a nearly vertical path at its terminal velocity.

Under this condition there is a moment on the wings, giving rise to a downward load on the front truss and an upward load on the rear truss, the wings being at negative angle of attack.

The machine is kept in equilibrium by a load acting down on the tail.

At a steady velocity the weight is balanced by the drag on the various parts.

In pulling out of a nose dive heavy loads are put on a machine. A quick pull out is liable to break a machine, as shown by the following example.

An aeroplane nose-diving at a speed of 250 m.p.h. is suddenly pulled out and reaches an angle of attack equal to the stalling angle without any appreciable loss of speed. If the stalling speed is 80 m.p.h., find the minimum load factor required to prevent failure of the structure.

Actual lift at stalling angle at 250 m.p.h.

$$= \frac{\frac{1}{2}C_L \rho A V^2}{g}$$

Normal lift at stalling angle at 80 m.p.h. in steady horizontal flight

$$= \text{weight} = \frac{\frac{1}{2}C_L \rho A v^2}{g}$$

where V = speed at 250 m.p.h.

and v = speed at 80 m.p.h.

$$\frac{\text{Actual lift}}{\text{Normal lift}} = \frac{\text{actual lift}}{\text{weight}} = \text{load factor required.}$$

$$= \frac{gC_L \rho A V^2}{gC_L \rho A v^2} = \frac{V^2}{v^2} = \frac{250^2}{80^2}$$

$$= \underline{9{\cdot}8}$$

If the ultimate factor is not more than 9·8 the machine would fail under these conditions. If the ultimate factor is 12, the aeroplane may have permanent strain.

Loads in a Turn

When an aeroplane turns, the side loads on the fin and rudder put a horizontal load on the fuselage, and this must be taken into account, together with the vertical loads on the tail plane. Thus in a braced fuselage the top and bottom as well as the side bracings will be stressed.

If, as is usually the case, the fin and rudder are not symmetrically placed about the centre line of the fuselage, the fuselage has to withstand a torsion as well as the direct load.

Landing Loads

When a machine lands the shock is absorbed in the shock absorbers of the undercarriage and the tail wheel.

In a bad landing heavy loads are put on the fuselage, the rear portion of which acts as a cantilever to take the tail load.

The inertia of the wings causes loads in the main plane structure, which are not serious, unless engines or other weights are carried in the wings.

CHAPTER V

STRENGTH OF MATERIALS

VOLUME III in this series being devoted to Strength of Materials, this chapter will not give more than a brief outline of some of the more important principles involved in the structure of the aeroplane.

Strain

All bodies are altered in shape by the forces acting on them. This distortion is called Strain.

Stress

The internal forces in the material of a body that are called into play to resist strain are called Stress.

Most solids are elastic up to a certain limit; if strained beyond this limit and the stress removed, they will not return to their original shape.

This permanent change of shape is known as the Permanent Set.

Within the elastic limit of a material the ratio $\frac{\text{stress}}{\text{strain}}$ is a constant called Young's Modulus, or the Modulus of Elasticity of the material.

For all members taking pure tension or compression

$$\text{Stress} = \frac{\text{Load}}{\text{Cross-sectional area}}$$

$$\text{Strain} = \frac{\text{Increase or decrease in length}}{\text{Original length}}$$

A member in shear will have a shear stress equal to

$$\frac{\text{Load}}{\text{Area under shear}}$$

Bearing stress, which is an important consideration when thin material is used, is equal to

$$\frac{\text{Load}}{\text{Thickness} \times \text{Diameter}}$$

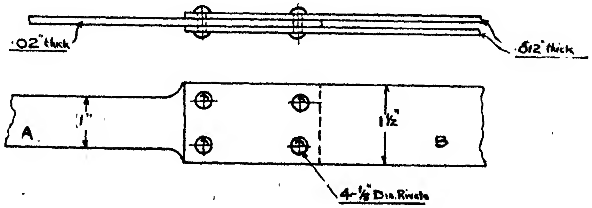


FIG. 29

Example. *If there is a tension of 1 ton in the member shown in Fig. 29, find—*

(a) The maximum tensile stress in A.
(b) The maximum tensile stress in B.
(c) The shear stress in the rivets.
(d) The maximum bearing stress.

(*a*) Tensile stress in *A*

$$= \frac{\text{Load}}{\text{Area}} = \frac{1}{0{\cdot}02 \times 1}$$

$$= \underline{50} \text{ tons/sq. in.}$$

(*b*) Load in each side

$$= \tfrac{1}{2} \text{ ton.}$$

Cross-section area at point of maximum stress

$$= (\text{width of strip—diam. of two rivets}) \text{ thickness}$$

$$= (1\tfrac{1}{2} - \tfrac{1}{4})\, 0{\cdot}012$$

$$= 1{\cdot}25 \times 0{\cdot}012 \text{ sq. in.}$$

Maximum stress

$$= \frac{0{\cdot}5}{1{\cdot}25 \times 0{\cdot}12}$$

$$= \underline{33\tfrac{1}{3}} \text{ tons/sq. in.}$$

(*c*) Area of rivets under shear

$$= 8 \times \text{cross-sectional area of rivets}$$

$$= \frac{8\pi(\frac{1}{8})^2}{4}$$

$$= \frac{8 \times 0{\cdot}7854}{64}$$

$$\text{Stress} = \frac{\text{load}}{\text{area}}$$

$$= \frac{1 \times 64}{8 \times 0{\cdot}7854}$$

$$= \underline{10{\cdot}2} \text{ tons/sq. in.}$$

(*d*) Total bearing area

$$= \text{thickness of plate} \times 4 \times \text{diam. of rivets.}$$

$$= 0{\cdot}02 \times 4 \times \tfrac{1}{8}$$

$$\text{Stress} = \frac{1}{0{\cdot}02 \times 4 \times \frac{1}{8}}$$

$$= \underline{100} \text{ tons/sq. in.}$$

Struts

Owing to the need for greater accuracy in aeroplane work, it has been found necessary to modify existing strut formulae to take into account the equivalent eccentricity of loading due to very small deviations from straightness and variations of thickness which in practice must exist in all struts.

All compression struts, including longerons, are considered to be pin-jointed members for purposes of stressing, as although they may not be pin-jointed their fixings are not rigid.

For long struts of uniform section where the length/radius of gyration is greater than 130, Euler's formula—

$$\text{Buckling load, } P = \frac{\pi^2 EI}{l^2}$$

is sufficiently accurate.

As, however, by far the greater number of struts have a l/K ratio of less than 130, it is necessary to use a much more complicated formula, due to Southwell, viz.—

$$\text{Limiting load } P = pA = \frac{p_y A}{1 + \frac{d\delta}{2K^2}\,\text{Sec.}\left\{\sqrt{\frac{p}{EK^2}} \times \frac{l}{2}\right\}}$$

where P = limiting load or load which makes the maximum stress equal to the yield stress

p = limiting average stress

A = area of cross-section

p_y = yield point

d = depth of section in plane of bending

= outside diameter of circular tube

K = minimum radius of gyration of cross-section

E = Young's Modulus

l = length of strut

δ = equivalent eccentricity of loading, due to crookedness of tube and eccentricity of the bore of the tube

For tubular struts this equals

$$\frac{l}{600} + \frac{\text{Internal diameter}}{40}$$

This formula cannot be used directly for finding the size of a strut, but values of P or p may be found graphically or by trial and error.

It is usual to draw a graph having p and l/K as ordinates. From this curves may be drawn for tubes of standard thickness and diameter with P and l as ordinates.

Examples of these graphs and tube constants will be found in Appendix III.

Example 1. Three steel tubes are required of lengths, 2 ft., 3 ft., and 4 ft., to take compressive end loads of 8500 lb. each. If the steel used has a yield stress of 40 tons/sq. in. and the struts are to be 17 S.W.G. thick, find the diameter in each case.

Referring to the strut curves for 17 S.W.G. 40 yield steel in Appendix III, the diameters may be found directly from the graphs.

2 ft. strut requires $1\frac{1}{8}$ in. diam.
3 ft. ,, ,, $1\frac{3}{8}$ in. ,,
4 ft. ,, ,, $1\frac{5}{8}$ in. ,,

Example 2. Find the diameters and weights of three 20 S.W.G. tubes, each 3 ft. 6 in. long and carrying a compressive load of 4400 lb.

They are made up of (*a*) duralumin, (*b*) 28 ton yield steel, and (*c*) 45 ton yield steel.

Weight of steel $= 0{\cdot}283$ lb./cub. in.
Weight of duralumin $= 0{\cdot}1$ lb./cub. in.

Referring to Appendix III—

(*a*) Diam. of duralumin strut $= 2$ in.
Area $= 0{\cdot}222$ sq. in.
Weight $= 0{\cdot}222 \times 42 \times 0{\cdot}1$
$= \underline{\underline{0{\cdot}932}}$ lb.

(*b*) Diam. of 28 ton steel strut $= 1\frac{1}{2}$ in.
Area $= 0{\cdot}1655$ sq. in.
Weight $= 0{\cdot}1655 \times 42 \times 0{\cdot}283$
$= \underline{\underline{1{\cdot}97}}$ lb.

(*c*) Diam. of 45 ton steel strut $= 1\frac{3}{8}$ in.
Area $= 0{\cdot}151$ sq. in.
Weight $= 0{\cdot}151 \times 42 \times 0{\cdot}283$
$= \underline{\underline{1{\cdot}795}}$ lb.

Example 3. A $1\frac{1}{2}$ in. diam. 24 S.W.G. tube is made in steel having a yield point of 65 tons/sq. in.

Find the maximum compressive load it will withstand on a length of 25 in.

Referring to Appendix III—

K for $1\frac{1}{2}$ in. diam. 24 S.W.G. tube $= 0{\cdot}523$.

$$l/K = \frac{25}{0{\cdot}523} = 47{\cdot}8.$$

From graph—

Limiting stress $= 38$ tons/sq. in.
Area of tube $= 0{\cdot}102$ sq. in.
Limiting load $= 38 \times 0{\cdot}102$
$= 3{\cdot}88$ tons
$= 3{\cdot}88 \times 2240$
$= \underline{\underline{8700}}$ lb.

It should be noted that for a given load, any increase in length will mean an increase in area and weight, and that a short length with a big load may be the same area as a long length with a small load. This is an important consideration in determining the length of bays where a continuous member such as a longeron is used.

Within limits increase in diameter of the tube will mean a decrease in weight.

Comparing tubes made of 28 tons/sq. in. yield steel, required to carry a compressive load of 4000 lb. on a length of 50 in., the following results, showing a considerable save in weight with the larger diameters, may be found from Appendix III.

Diameter	Gauge	Area	Weight (lb./ft.)
1¼ in.	17	0·232	0·843
1½ in.	20	0·166	0·624
1¾ in.	22	0·135	0·533

Choosing large diameters leads to a difficulty, inasmuch as the ratio of the diameter to thickness is getting large. Besides there being a commercial limit to the thinness of the walls, the strut will buckle before yield stress of the material is developed should diameter/thickness exceed 100, due to elastic instability. It follows, therefore, that when this limit is reached, any increase in diameter must be accompanied by a proportional increase in thickness, the weight increasing as the square of the diameter.

The above may be demonstrated by taking two short cylinders of drawing paper, one of 2 in. diameter and the other 3½ in. diameter. Load them with small books, and it will be found that the 2-in. diameter strut is stronger.

These difficulties may be overcome by the use of high-grade steel strip, which after appropriate corrugation is assembled into tubular members as illustrated in Fig. 30 (b).

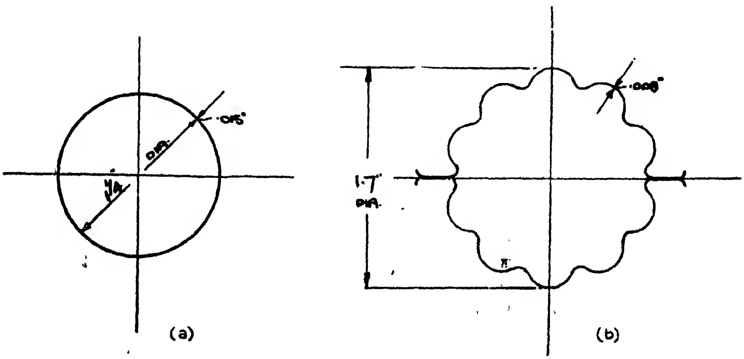


FIG. 30

Comparing a 36 in. length of the tube (Fig. 30 (a)) of 45 tons/sq. in. yield, with the same length of the corrugated specimen having a 65 ton/sq. in. yield point, it is found that case (a) would fail under a

load of 2000 lb. while the built-up tube, although being 15 per cent lighter, would sustain a load of 2600 lb. before failure.

Fig. 31 gives a comparison showing the increased stress a tube will stand with increased corrugation.

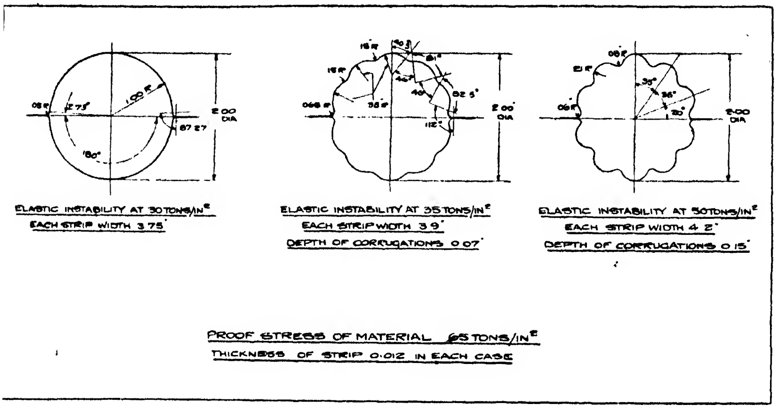


Fig. 31

Beams

Any structure acted upon by forces oblique to its longitudinal axis is called a Beam.

A beam has to withstand shear and bending.

Shear

The Shear force at any section of a beam is equal to the algebraic sum of the forces resolved normal to the beam on either side of the section considered.

The shear stress varies across the section, being greatest at the neutral axis.

Maximum shear stress at any section $= \frac{FAz}{It}$ (See Chap. XI, Vol. III.)

where F = shearing force at section considered
I = moment of inertia of section
A = area of section above neutral axis
z = distance of centroid of half section A above neutral axis
t = total thickness of web or webs at neutral axis

If the beam is a braced structure such as a fuselage, or rib, the shear will be taken as tension and compression in the bracing members.

Bending. The stresses due to bending are not directly dependent on the forces acting on the beam, but on the moments of these forces.

The tendency for a beam to bend at any point due to the moments of the external forces is called the Bending Moment, and must be resisted by an equal and opposite moment or couple exerted by the material of the beam at that section. This couple, called the Moment of Resistance, is formed by a tensile reaction on one side and a compressive reaction on the other side of the beam.

The Bending Moment is equal to the algebraic sum of the moments of all the forces on either side of the section considered.

The variation of bending moment and shear force on a beam may be shown by plotting values of bending moment and shear force along the length.

EXAMPLE 1. Draw bending moment and shear force diagrams for the beam loaded as in Fig. 32.

To find reactions R_1 and R_2, take moments about A.

$$4 \times 6 + 9 \times 4 + 16 \times 6 - 12R_2 = 0$$

$$R_2 = \frac{24 + 36 + 96}{12}$$

$$= \frac{156}{12}$$

$$= \underline{\underline{13}}$$

$$R_1 = 6 + 4 + 6 - 13$$

$$= \underline{\underline{3}}$$

Taking upward forces from the left positive,

Shear forces between A and B = 3 tons.

" " " B " $C = 3 - 6 = -3$ tons.

" " " C " $D = 3 - 6 - 4 = -7$ tons.

" " " D " $E = 3 - 6 - 4 + 13 = 6$ tons.

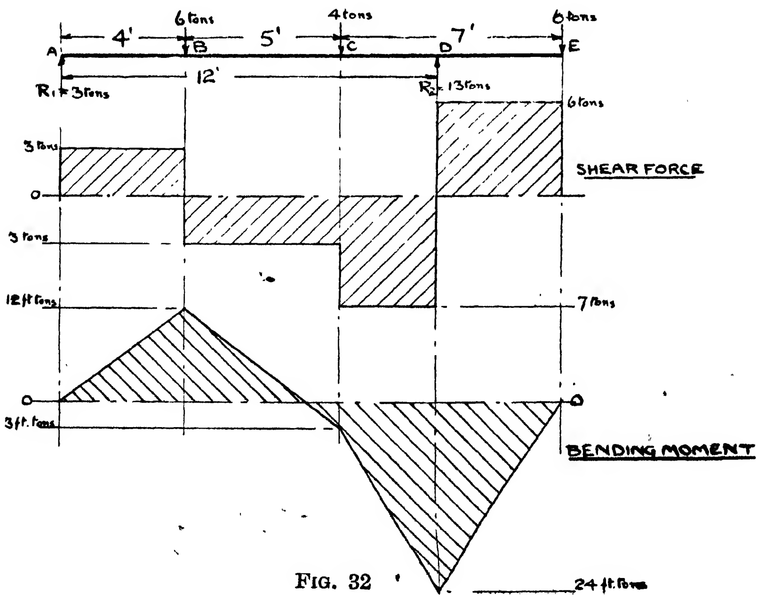


FIG. 32

Taking clockwise moments from the left positive,

Bending moment at $A = 0$.

,, ,, $B = 4 \times 3 = 12$ ft. tons.

,, ,, $C = 9 \times 3 - 5 \times 6 = -3$ ft. tons.

,, ,, $D = 12 \times 3 - 8 \times 6 - 3 \times 4 = -24$ ft. tons.

,, ,, $E = 16 \times 3 - 12 \times 6 - 7 \times 4 + 4 \times 13 = 0$.

As the bending moment is proportional to the distance, the diagram will be a straight line between the points of application of the load.

EXAMPLE 2. Draw the shear force and bending moment diagrams for the beam having a distributed load shown in Fig. 33.

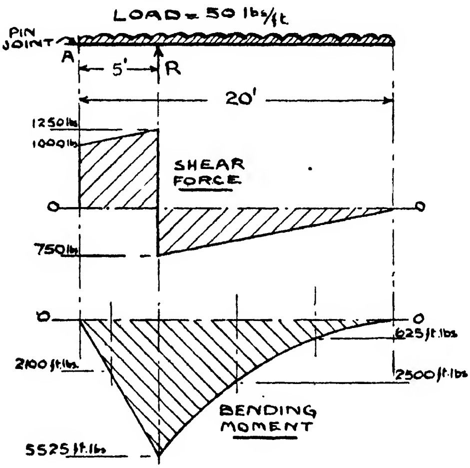


FIG. 33

$$\text{Total load} = 20 \times 50$$
$$= \underline{1000 \text{ lb.}}$$

To find R, take moments about A.

$$1000 \times 10 - 5R = 0$$
$$R = \frac{10000}{5}$$
$$= \underline{2000 \text{ lb.}}$$

Shear force at extreme right $= 0$. This increases uniformly to

$$-50 \times 15 = -750 \text{ lb. at } R.$$

At R it equals $2000 - 750 = 1250$ lb., and then decreases uniformly to 000 lb. at A.

Bending moment at extreme right = 0.
Bending moment at point 5 ft. from right

= load on 5 ft. × distance of C.G. of load on 5 ft. from right
= 5 × 50 × 2·5
= 625 ft.-lb.

At 10 ft. from right

B.M. = 10 × 50 × 5
= 2500 ft.-lb.

At R, B.M. = 15 × 50 × 7·5
= 5525 ft.-lb.

At 18 ft. from right

B.M. = 18 × 50 × 9 − 2000 × 3
= 8100 − 6000
= 2100 ft.-lb.

At A B.M. = 20 × 50 × 10 − 2000 × 5
= 0.

Theory of Bending

In dealing with simple bending, the following assumptions are made—

1. A plane cross-section at right angles to the plane of bending always remains in a plane.

2. The Modulus of Elasticity of the material is the same for tension and compression.

3. The material is free to expand or contract longitudinally and laterally as if it were made of separate layers.

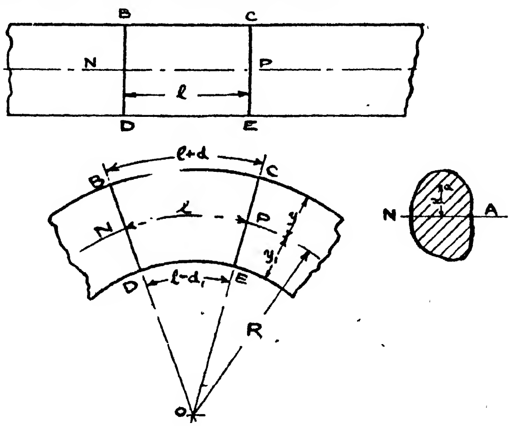


Fig. 34

4. The material obeys Hooke's Law, i.e. the stress is proportional to the strain.

It is important that these assumptions should be known, as in some cases, such as reinforced concrete beams, they do not hold good. They are, however, sufficiently accurate to be applied to all beams used in aircraft at the present time.

Neutral Axis

Referring to Fig. 34, let BD and CE be two parallel cross-sections of a beam, distance l apart. On bending they will no longer be parallel, BC having increased in length, and DE decreased. There must be a plane where the material remains the same; this plane NP is called the Neutral Plane, and its line of intersection with the cross-section of the beam is called the Neutral Axis (NA). This neutral axis passes through the centroid of the section.

Stresses in Beams

Let f = the max. stress in the material at the section considered
y = the distance of the further outside of the section from the N.A.
M = the bending moment at the section
I = the moment of inertia of the area of the section about the N.A.
E = modulus of elasticity of the material
R = radius of curvature of the beam at the N.A.

Consider the length of l on the neutral plane of the bent beam. At a distance y outwards it has increased to $l + d$, and at a distance y, inwards it has decreased to $l - d_1$.

If the section is symmetrical about N.A.,

$$y = y_1 \text{ and } d = d_1$$

$$\frac{BC}{CO} = \frac{NP}{PO}$$

i.e.

$$\frac{l + d}{R + y} = \frac{l}{R}$$

$$Rl + Rd = Rl + yl$$

$$Rd = yl$$

$$\frac{d}{l} = \frac{y}{R}$$

but

$$\frac{d}{l} = \frac{\text{max. increase in length}}{\text{original length}} = \text{max. strain.}$$

$$\frac{\text{Stress}}{\text{Strain}} = E, \text{ or strain} = \frac{f}{E}$$

$$\therefore \quad \frac{f}{E} = \frac{y}{R} \text{ or } \frac{f}{y} = \frac{E}{R}$$

Now consider a small element of area a, distance x from N.A., subjected to a stress of f_1.

The total force in this area $= f_1 a$ $\left(\text{stress} = \frac{\text{load}}{\text{area}}\right)$ and its moment about N.A. $= f_1 a x$.

The total moment about N.A. = Moment of Resistance = the sum of the moments of all the elements of area which make up the whole cross-section = $\Sigma f_1 ax$.

Moment of resistance $= M$.

$\therefore \quad M = \Sigma f_1 ax$.

We may represent the variation of strain by the diagram in Fig. 35.

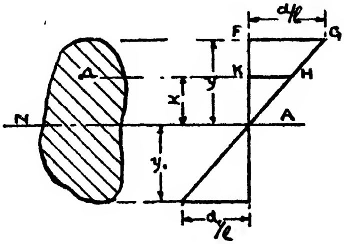


FIG. 35

The stress being proportional to the strain both for tension and compression, this diagram will also show the stress distribution.

If FG represents f, then KH will represent f_1.

From similar triangles

$$\frac{f}{y} = \frac{f_1}{x}$$

$\therefore$
$$\frac{f_1}{x} = \frac{E}{R}$$

and
$$f_1 = \frac{E}{R} x.$$

From above
$$M = \Sigma f_1 ax.$$

Substituting for f_1
$$M = \frac{E}{R} \Sigma ax^2.$$

But
$$\Sigma ax^2 = I,$$

$\therefore$
$$M = \frac{E}{R} I$$

or
$$\frac{M}{I} = \frac{E}{R} = \frac{f}{y}.$$

This is an important relation from which we get $M = \frac{f}{y} I$, sometimes written $M = fZ$, where Z is a constant depending on the shape and size of the section, called the Modulus of Section, and equal to $\frac{I}{y}$.

The Effect of the Shape of the Section

In Fig. 36 the strength of an "I" Section beam is compared with that of a rectangular one of equal area, weight, and depth, and the same material.

The maximum stress, and therefore the stress distribution, is the same in each case.

The internal force or load distributions over the sections are found by dividing the sections into small strips and finding the products of their respective areas and stresses.

In this case the width of each strip is taken as 0·1 in.

The area of each strip of the rectangle is therefore 0·1 sq. in.

Load/Strip = Stress × Area of strip.

This decreases uniformly from 1 ton tension at the top to zero at the centre and 1 ton compression at the bottom.

In the "I" Section the areas will be 0·2 sq. in. in the flanges and 0·05 sq. in. in the web. This gives the load distributions shown on the right.

The total loads in each case are the product of the average load/strip and the number of strips, and may be assumed to act through the centroid of the load graph.

Thus the rectangular beam has a resistance of 7·5 tons tension and compression acting at 2 in. from each other, and the moment of resistance will equal 2 × 7·5 = 15 in.-tons.

Whilst the "I" beam gives resistance of 10 tons at $2\frac{1}{3}$ in., and its moment of resistance will equal $2\frac{1}{3} \times 10 = 23\frac{1}{3}$ in.-tons.

I.e. the "I" Section will withstand $8\frac{1}{3}$ in.-tons extra bending moment.

The above should make it clear that by putting the bulk of the material at a distance from the neutral axis where the stress is a maximum—

(*a*) The total resisting force is increased.
(*b*) The distance of its line of action is increased.
(*c*) The moment of resistance is increased.
(*d*) The strength/weight ratio is increased.

In the same way it may be seen that by increasing the depth of a beam—

(*a*) The total resisting force is unaltered.
(*b*) The distance of its line of action is increased.
(*c*) The moment of resistance is increased.
(*d*) The strength/weight ratio is increased.

Values of I and y for various standard sections, and a method of finding I for a rolled metal section, are given in Appendices I and II.

EXAMPLE. In the beam illustrated in Fig. 37, find the maximum stress due to bending and shear. Neglect the weight of the beam.

Taking moments about A,

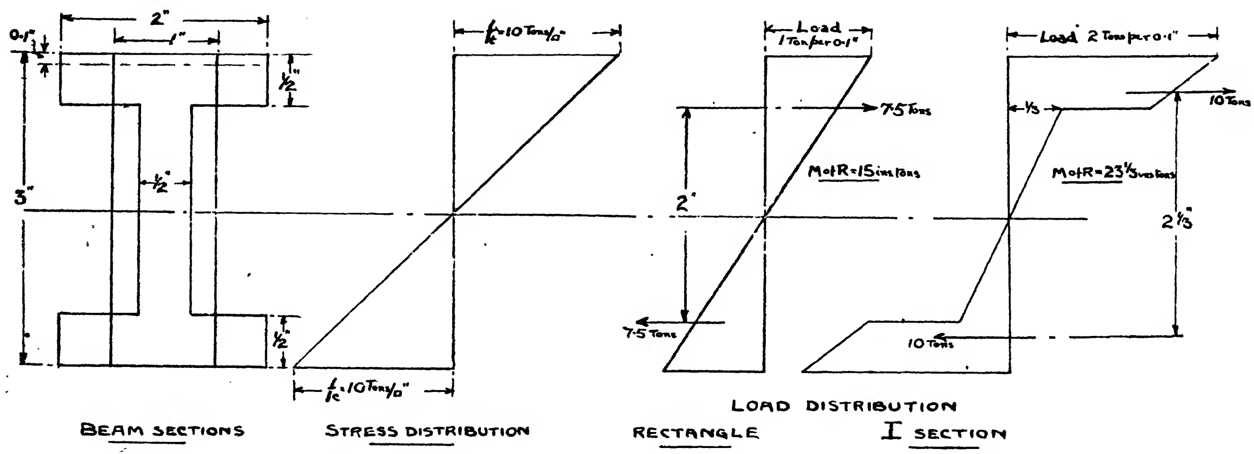
$$6 \times 1200 + 14 \times 800 - 20R_1 = 0$$

$$R_1 = \frac{7200 + 11200}{20}$$

$$= \underline{920 \text{ lb.}}$$

$$R_2 = 1200 + 800 - 920$$

$$= \underline{1080 \text{ lb.}}$$



2"
0·1"
1"
½"
3"
½"
½"
f_t = 10 Tons/□"
f_c = 10 Tons/□"
Load 1 Ton per 0·1"
7·5 Tons
Mo f R = 15 ins tons
2"
7·5 Tons
Load 2 Tons per 0·1"
⅓
10 Tons
Mo f R = 23⅓ ins tons
2⅓"
10 Tons
BEAM SECTIONS
STRESS DISTRIBUTION
LOAD DISTRIBUTION
RECTANGLE
I SECTION

FIG. 36

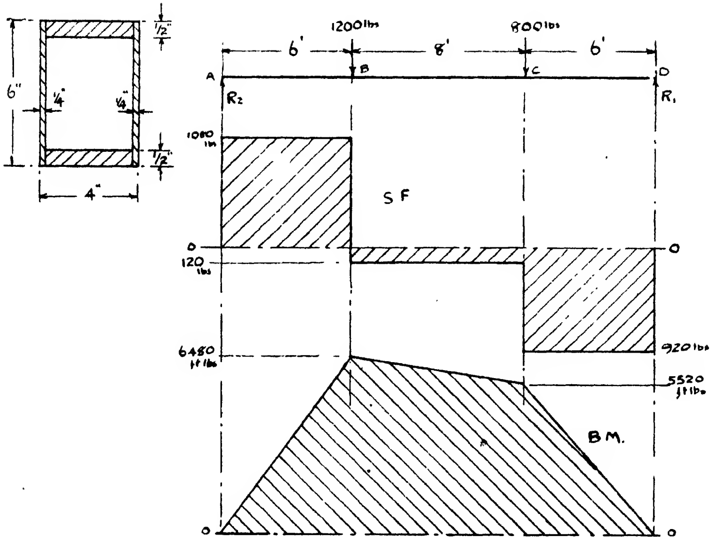


FIG. 37

Shear force

between A and B = 1080 lb.,

" B and C 1080 − 1200 = − 120 lb.,

" C and D 1080 − 1200 − 800 = − 920 lb.

Draw the shear force diagram.

Maximum shear force = 1080 lb.

$$\text{Shear stress} = \frac{FAz}{It}$$

$$A = \tfrac{1}{2} \times 3\tfrac{1}{2} + \tfrac{1}{2} \times 3$$
$$= 1\tfrac{3}{4} + 1\tfrac{1}{2}$$
$$= \underline{\underline{3\tfrac{1}{4}}} \text{ sq. in.}$$

To find z, take moments about the neutral axis.

$$\tfrac{1}{2} \times 4 \times 2\tfrac{3}{4} + \tfrac{1}{2} \times 2\tfrac{1}{2} \times 1\tfrac{1}{4} - 3\tfrac{1}{4}z = 0.$$

$$z = \frac{5{\cdot}5 + 1{\cdot}563}{3{\cdot}25}$$
$$= \underline{\underline{2{\cdot}17 \text{ in.}}}$$

$$I = \frac{b_1d_1^3 - b_2d_2^3}{12}$$
$$= \frac{4 \times 6^3 - 3\tfrac{1}{2} \times 5^3}{12}$$
$$= \frac{426{\cdot}5}{12} = \underline{\underline{35{\cdot}54 \text{ in.}^4}}$$

Substituting in above formula

$$\text{Max. shear stress} = \frac{1080 \times 3{\cdot}25 \times 2{\cdot}17}{35{\cdot}54 \times 0{\cdot}5}$$
$$= \underline{42{\cdot}9} \text{ lb./sq. in.}$$

Bending moment at $A = 0$.
" " $B = 6 \times 1080 = 6480$ ft.-lb.
" " $C = 14 \times 1080 - 8 \times 1200 = 5520$ ft.-lb.
" " $D = 0$.

Draw bending moment diagram.

Max. bending moment is at $B = 6{,}480$ ft.-lb.
$= 6480 \times 12$ in.-lb.

$$M = \frac{fI}{y}$$

$$\text{Max. stress due to bending } f = \frac{My}{I}$$
$$= \frac{6480 \times 12 \times 3}{35{\cdot}54}$$
$$= \underline{6560} \text{ lb./sq. in.}$$

Effect of End Load

The spars of an aeroplane are not only subjected to bending by the lift reactions, but in addition have to bear heavy end loads due to the type of bracing used.

The total stress will be equal to the stress due to bending plus the direct stress due to the end load

$$f = \frac{My}{I} + \frac{P}{A}$$

where P = end load and A = area of cross-section.

The end load has the effect of increasing or decreasing the bending moment (M), depending on whether it is compression or tension.

Referring to Fig. 38, case (a) shows a beam simply loaded at the centre; the maximum bending moment will be at the centre and equal to $\frac{Wl}{4}$.

In case (b) let x_1 be the deflection at the centre. Then, taking moments on one side of the centre, the bending moment at this point

$$= \frac{Wl}{4} + Px_1$$

i.e. M has increased by Px_1.

Case (c) has a tensile end load, and in the same way

$$M = \frac{Wl}{4} - Px_2$$

showing M has decreased by Px_2.

x_1 is greater and x_2 less than x, due to the increased and decreased bending moments respectively.

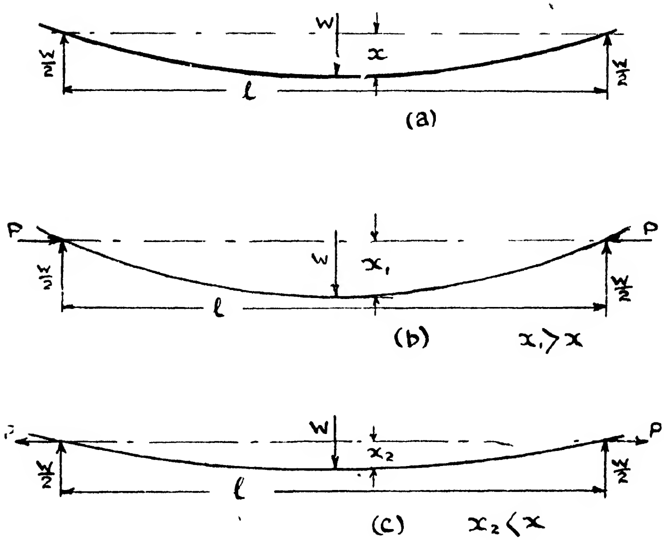

W
x
W/2
l
(a)
P
W
x_1
(b)
$x_1 > x$
x_2
(c)
$x_2 < x$

FIG. 38

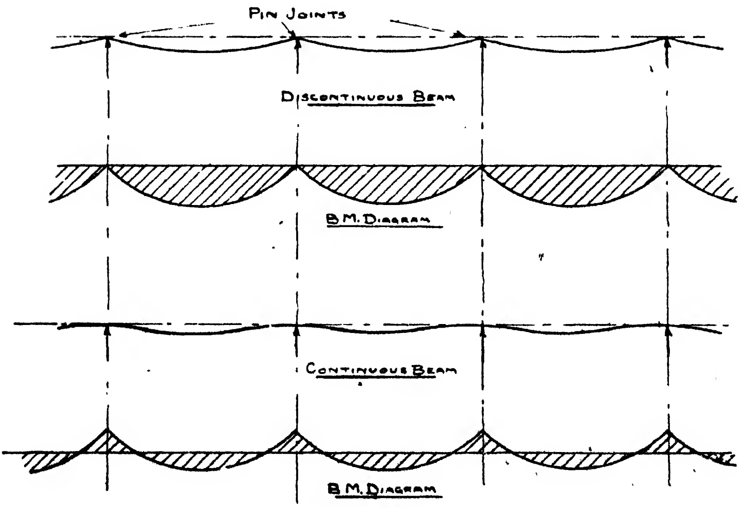

PIN JOINTS
DISCONTINUOUS BEAM
B.M. DIAGRAM
CONTINUOUS BEAM
B.M. DIAGRAM

FIG. 39

The above results may be very easily illustrated by taking a long ruler loaded at the centre, and, using the hands as supports, first compress it and then put a pull in it. It will be seen that the deflection or strain, and therefore the bending stress, is increased in the first instance and decreased in the second.

It will be seen from the foregoing that a beam with a compressive end load will have to be made much heavier than a similar one having a tensile or no end load.

This is a very important consideration in determining the arrangement of bracing.

The continuity of the spars and the end loads in them render the evaluation of the stresses and the reactions at the supports a too difficult mathematical operation for inclusion in this book.

It may be noted by reference to Fig. 39 that a continuous beam will develop less deflection and therefore less stress, other things being equal, than a discontinuous one.

Thin Metal Sections

In order that spars and other beams and compression members made from metal strip may have a high strength/weight ratio, it is necessary—

1. To have the bulk of metal as far as possible from the neutral axis.
2. Design the spar so that it will be elastically stable.

The depth of a spar will depend on the wing section used. In order that it may pass inside the rib booms, it will have to be approximately 1 in. less in depth than the aerofoil at the centre line of the spars.

The farther away the material is from the neutral axis, the greater will be the moment of inertia, and therefore the greater its power of resisting bending and compression.

For this reason the webs are often made of smaller gauge than the flanges, if by so doing the shear stress is not exceeded.

The maximum stress will be at the parts farthest away from the neutral axis, and it will be necessary to take particular care to see that the member is elastically stable at these parts.

Elastic Instability

A member fails by elastic instability if, by the formation of a "wave" or local buckle due to lack of rigidity, failure is caused before the elastic limit of the material is reached.

This may be simply illustrated by taking two pieces of paper, folding one into a channel section, and corrugating the other as in Fig. 40 (*a*) and (*b*). It is obvious that the least radius of gyration of the corrugated specimen is the smaller, and that, apart from elastic instability, this would fail at a smaller load.

Press very lightly on the channel, and it will be found to form waves and fail. Do the same with the other piece, and it will be found to stand a comparatively large load.

In the first case the failure takes place due to elastic instability, while in the second a much higher stress was reached, the member having been made more rigid due to the corrugations.

If we refer back to the Euler formula for struts, we see that the strength of a long slender strut does not depend on the strength of the material used, but on its elasticity. Now E is practically the same for

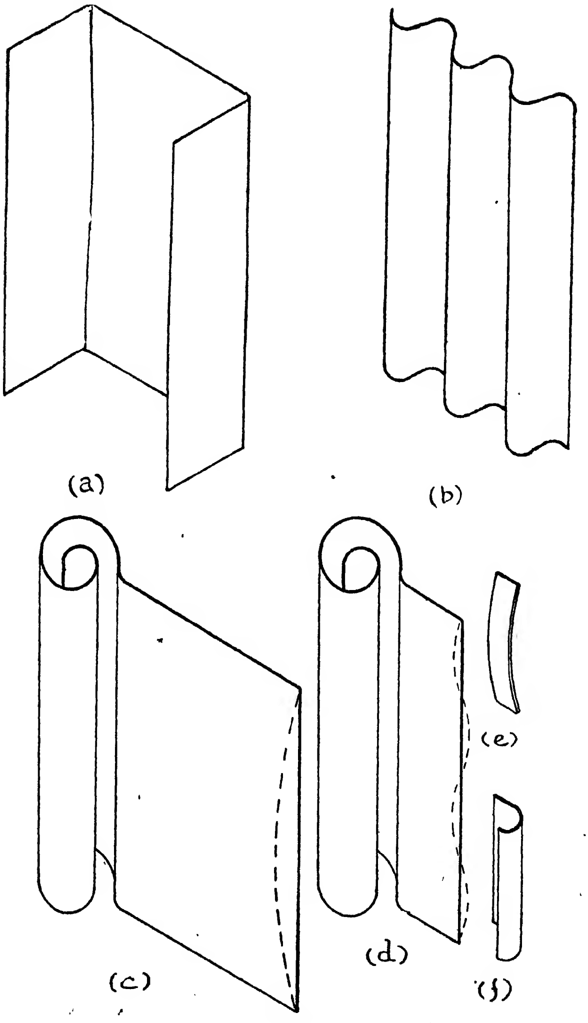


Fig. 40

all steels, so that in the case of a long slender strut, a high-tensile steel would be no stronger than mild steel.

Take a block of rubber, about 1 ft. long and 2 in. × 1 in. cross-section, stand it on end and apply a load by the hand. It will be found to withstand only a few pounds. Now think what a large load a similar strut

of a fairly weak wood will withstand, yet wood is not a very much stronger material than rubber. Due to the very low value of E, the rubber deflected a lot for a small load and this gave a large bending moment which increased with deflection more rapidly than the moment of resistance of the strut increased. The cause of failure of any slender strut is similar, and called elastic instability.

The stress at which elastic instability occurs depends on the slenderness of the strut and the Modulus of elasticity of its material. If elastic

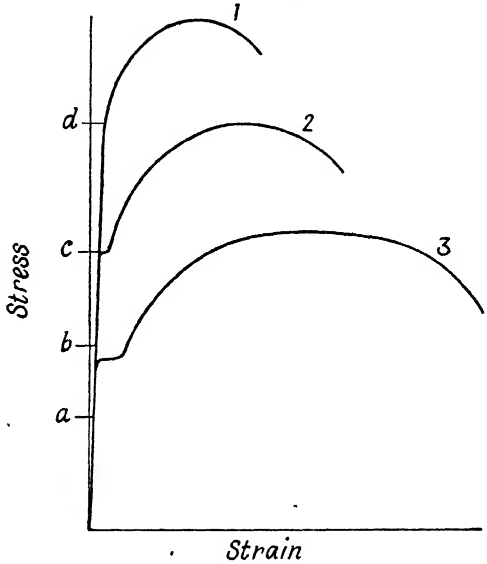


FIG. 40 (g)

instability does not occur until the elastic limit of the material is reached, it does not matter, as the full working strength of the material will have been reached, but if it occurs below the elastic limit the full strength cannot be taken advantage of.

Consider three struts 1, 2 and 3 made of materials giving the stress/strain curves shown in Fig. 40 (g). If the strut be such that elastic instability occurs at stress (a) struts 1, 2 and 3 will not develop the full strength of the material used. If elastic instability occurs at stress (b) strut 3 will be satisfactory, strut 2 will be slightly stronger, but will not take full advantage of its better material, whilst strut 1 will fail at the same load as strut 2 although made of stronger material. For strut 2 to be satisfactory it must develop stress (c) before becoming elastically unstable, and strut 1 must develop stress (d) to take advantage of the strong material used.

We may have a compression member which is not slender, when considered as a whole, but will still fail by elastic instability within the

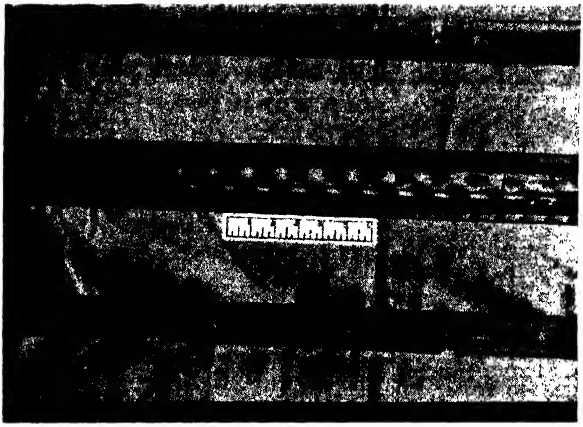


Fig. 41. Compression Failure of Spar Flange by Elastic Instability

Fig. 42. Wagner Beam under Shear

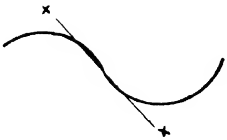


FIG. 43

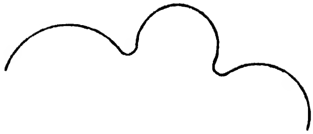


FIG. 44

FIG. 45

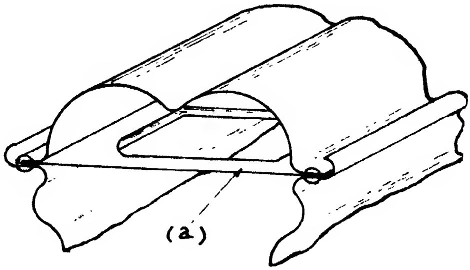


FIG. 46

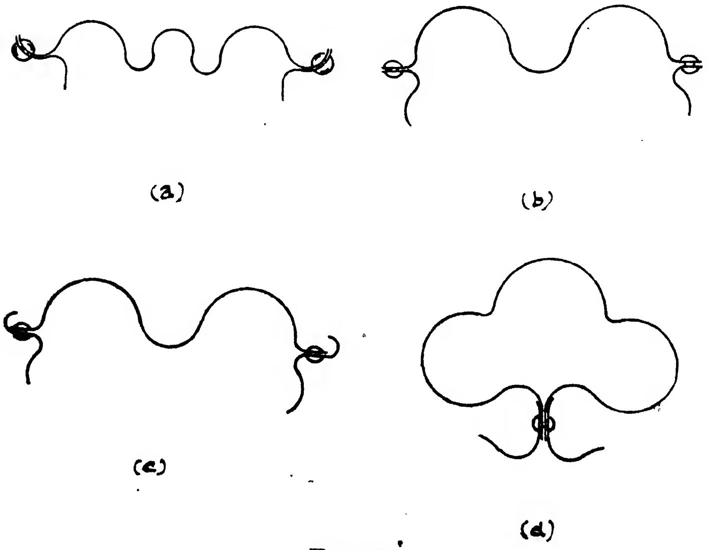


FIG. 47

elastic limit, e.g. the channel section is not less slender than the corrugated section Fig. 40, yet it failed first.

Consider the thin steel strut Fig. 40 (*c*). By normal strength calculations this will apparently withstand a much higher load than the actual buckling load. Long before the calculated load is reached, the outside edge will bow independently of the rest (shown dotted), and become elastically unstable. This bow will spread and cause a crinkle which will make the strut fail at a low stress.

If now the non-rigid portion is shortened (Fig. 40 (*d*)), the rigid part will prevent a single bow, but it will form the waves shown dotted; each wave being in effect a small slender elastically unstable strut. If the non-rigid portion is further shortened the waves will get still shorter.

Now imagine a portion the length of a wave cut out as in Fig. 40 (*e*), and we see it is an elastically unstable strut. As the waves get shorter these little "independent" struts become less slender, as the length-thickness ratio is getting smaller, until they become elastically stable. We see then that if the straight portion is sufficiently narrow, elastic instability will not occur.

If instead of making the straight portion narrower, a small radius is put down the outer edge of the strut in Fig. 40 (*d*), elastic instability will not occur, as the previous waved parts now form stable struts (Fig. 40 (*f*)).

From the above it will be seen that a thin metal strut or beam must be so corrugated that no portion can fail as a slender strut independently of the rest, if the full strength of the material is to be taken advantage of.

Fig. 41 shows a spar flange forming into waves, due to the compression stress being that at which elastic instability occurs. If the stress is reduced the waves will disappear, but if increased the waves will spread and the spar fail.

In order that a thin metal member may be elastically stable, special care must be taken in its design, especially in the spar flanges where the highest stresses occur.

Although many of the following points do not arise where light alloys are used, due to the comparative thickness of the material used, they are very important where thin high-grade steel is concerned.

Generally speaking, the following rules should be adhered to—

No flat portion should exceed eighteen times the thickness of the metal.

No radius should exceed thirty times the thickness. If two large radii are adjacent, there will be a "flat" where they join (Fig. 43 *xx*). This would have the same effect as too large a radius. It is therefore necessary at highly stressed parts to put a small radius next to a large one, as in Fig. 44. There must be enough of the small radius or it will have the effect of a very large radius (Fig. 45).

A flange may tend to flatten out under load; this may increase radius/thickness ratio to over thirty. This must be prevented by some method such as that illustrated in Fig. 46 (*a*), where light tension strips are fixed between the flange and webs.

Edges of metal should not be at points of high stress, as Fig. 47 (*a*) and (*b*), or the lip will fail independently of the rest of the spar. Under load it would form into a wave and crinkle, and this crinkle would spread across the spar, causing collapse at a stress considerably less than the yield stress of the material. (*b*) would be O.K. if it had small

radii at the lips as in (*c*). (*d*) will be all right as the ends are at points of low stress.

Wagner Beam

It often happens that only a very thin web is required to take the shear forces in a beam, and as in aeronautical engineering we cannot afford to waste weight, a thin web is often used. When such a beam is made up of two flanges joined by a thin deep plate web, as illustrated in Fig. 48 (*c*), it is called a Wagner beam. Such a beam cannot be treated for shear by the method given on page 42.

Consider a square portion $ABCD$, taken from the web of such a beam, Fig. 48 (*a*), and let it be subjected to shear forces S on the faces AD and BC. It will be seen that it is not in equilibrium, and therefore there must be equal shear forces present on the faces AB and CD, as shown. This illustrates a well-known principle, that whenever a shear stress exists in a body there must also exist an equal shear stress at right angles.

Let the areas of the faces AB, BC, etc., equal a, then the shear stress $= S/a$.

Now consider the forces on the diagonal AC, and we see that the shear forces combine to subject AC to a tensile force $S\sqrt{2}$. As the area at AC is $a\sqrt{2}$ the tensile stress across AC is—

$$\frac{S\sqrt{2}}{a\sqrt{\;}} = \frac{S}{a} = \text{shear stress.}$$

In the same way it may be shown that there is an equal compressive stress across the diagonal BD.

When the web is thick these compressive and tensile stresses are withstood by the material, and the beam will only fail, by shear, when the ultimate shear stress of the material is reached. When, however, the web is thin it will be elastically unstable under compression, and this will cause it to act in a similar manner to a braced girder.

The braced girder, shown subjected to a load P in Fig. 48 (*b*), has compression and tension in its flanges due to bending. The shear force in the first panel is resisted by tension in AC and compression in BD. If however BD is a wire it will bow, taking practically no load. The structure will not collapse as AC keeps the frame rigid. Now consider the cross-bracing replaced by a thin plate web, and we have the Wagner beam, Fig. 48 (*c*). Shear now puts the web in tension across AC, and compression across BD. BD is elastically unstable, so the shear is resisted by tension across AC. The web will form waves as it cannot form a single bow, as did the bracing wire, due to the restriction of the flanges. These waves, illustrated in Fig. 42, are not a sign of failure; they are elastic and disappear when the load is removed, as did the bow in the bracing of the girder.

Stresses in Wagner Beams

Consider the Wagner beam, Fig. 48 (*d*), to be divided in two by the section X–X. Apply the method of sections, given in Chapter II. The portion of beam to the right of X–X is in equilibrium under the applied

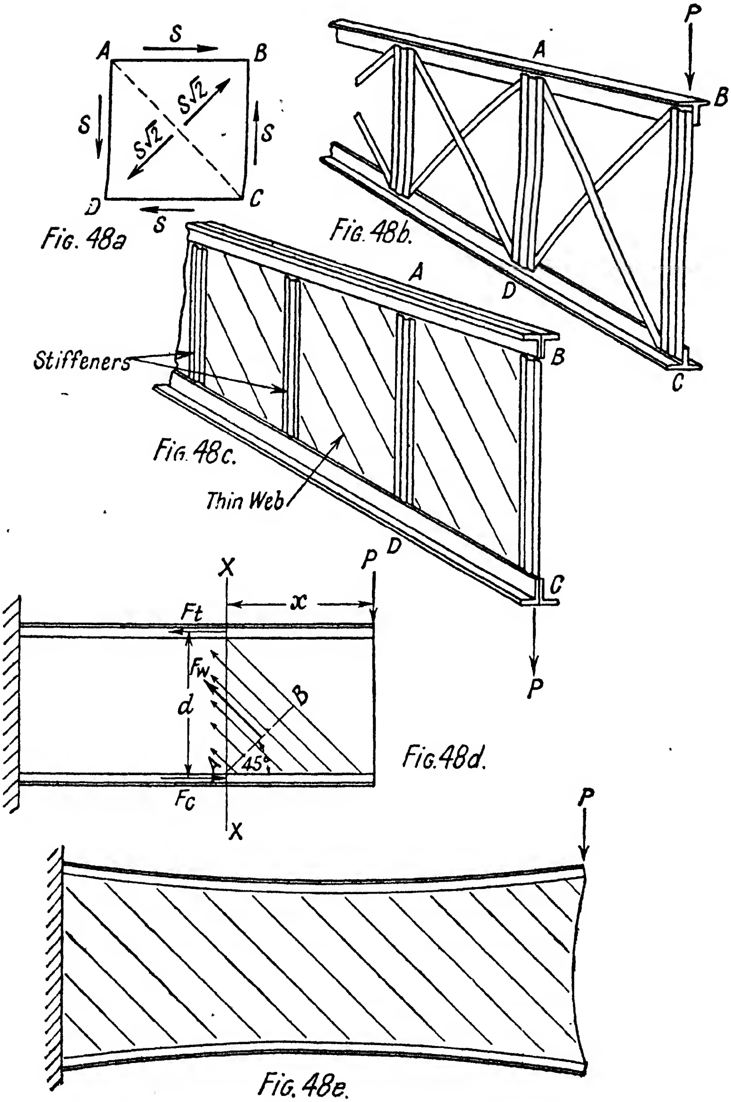

S
A
B
S
S√2
S√2
S
D
C
S
Fig. 48a
P
A
B
Fig. 48b.
D
C
A
Stiffeners
B
Fig. 48c.
Thin Web
D
C
P
X
P
x
Ft
Fw
d
B
45°
A
Fc
X
Fig. 48d.
P
Fig. 48e.

FIG. 48

force P, the tension in the top flange F_t, the compression in the bottom flange F_c, and the tension in the web F_w.

$$F_w = P/\sin 45° = P\sqrt{2}$$

The length of web AB on which it acts $= \dfrac{d}{\sqrt{2}}$

Taking moments about the bottom flange at X–X—

$$Px - F_t d - \frac{F_w d}{2\sqrt{2}} = 0$$

$$F_t = \frac{Px - \dfrac{F_w d}{2\sqrt{2}}}{d} = \frac{Px}{d} - \frac{F_w}{2\sqrt{2}}$$

$$= \frac{Px}{d} - \frac{P\sqrt{2}}{2\sqrt{2}} = \frac{Px}{d} - \frac{P}{2}$$

$$\text{i.e. } F_t = \frac{M}{d} - \frac{S}{2}$$

where M is the bending moment at the section, and S is the shearing force.

In the same way—
$$F_c = \frac{M}{d} + \frac{S}{2}$$

Stress in web $= \dfrac{F_w}{\text{area } A\text{–}B} = \dfrac{F_w}{td \sin 45°}$ (where t = thickness)

$$= \frac{\dfrac{P\sqrt{2}}{td}}{\sqrt{2}} = \frac{2P}{td} = \frac{2S}{td}$$

If there are no stiffeners the flange will bow inwards due to the pull of the webs, as shown in Fig. 48 (*e*). To prevent this stiffeners are fitted as shown in Fig. 48 (*c*), also the end member BC must be stiff.

$$\text{Maximum tensile stress in flange} = \frac{F_t}{A} + \frac{M_w}{Z}$$

where A = cross-sectional area of flange and M_w = bending moment due to web tension at stiffeners.

Treating the flanges as continuous beams over the stiffeners as supports and carrying a distributed load equal to the normal component of the web tension, which equals $\dfrac{s}{d}$ per unit length, the bending moment is a maximum at the stiffeners and equal to $\dfrac{Sl^2}{12d}$ where l = distance apart of stiffeners.

Thus maximum tensile stress in flange $= \dfrac{F_t}{A} + \dfrac{Sl^2}{12dZ}$

and maximum compressing stress in flange $= \dfrac{F_c}{A} + \dfrac{Sl^2}{12dZ}$.

Example. Find the maximum stress in the web of a Wagner beam with a 10 in. deep and 0·028 in. thick web, when loaded as the beam in Fig. 32.

Maximum shear force $=$ 7 tons.

$$\text{Stress in web} = \frac{2S}{td}$$

$$= \frac{2 \times 7}{0{\cdot}028 \times 10} = 50 \text{ tons/sq. in.}$$

Torsion

Consider the cylindrical bar (Fig. 48 (f)), having a force P applied at a distance R from the axis oo at one end, and fixed at the other.

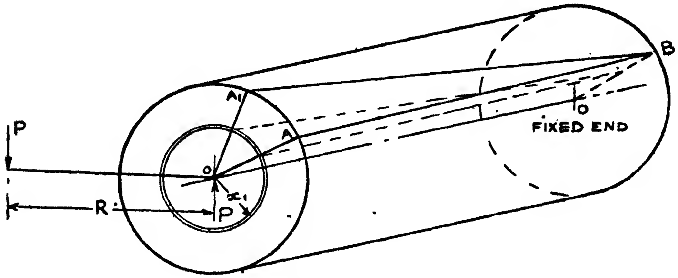


FIG. 48 (f)

There is a twisting moment or torque applied to it equal to PR, which must be resisted by an equal and opposite moment exerted by the material of the bar. This bar is said to be subjected to torsion.

The strain is such that the section at the loaded end will make a small rotation relative to the fixed end.

The line AB becomes A_1B under load, and the angle A_1BA is the shear strain at the surface.

This strain, and consequently (from Hooke's Law) the stress, will decrease uniformly from a maximum at the surface to zero at the axis, i.e. stress is proportional to distance from the centre.

Consider an elementary ring of radius x, and area a_1, and let the stress at this radius be f_1.

Let f be the maximum torsional stress in the bar, i.e. the stress at the surface, and y the distance of the surface from the axis.

Then
$$\frac{f_1}{f} = \frac{x_1}{y}$$

$$f_1 = \frac{fx_1}{y}$$

The force in the elementary ring $= f_1a_1$

Moment of resistance of elementary ring

$$= f_1a_1x_1$$

$$= \frac{f}{y}a_1x_1^2$$

Total moment of resistance = sum of moments of resistance of all elementary rings making up the whole cross-section

$$= \frac{f}{y}\,\Sigma a x^2$$

But $\Sigma a x^2$ = moment of inertia of section about axis *oo*.

Therefore moment of resistance $= \frac{f}{y}\,Ioo = PR =$ torque.

For a solid cylindrical bar $Ioo \doteq \frac{\pi}{32}\,D^4$

and $y = \frac{D}{2}$

where D = diameter.

Therefore

$$\text{torque} = \frac{f}{\frac{D}{2}}\,\frac{\pi}{32}\,D^4 = \underline{\frac{\pi}{16}\,fD^3}$$

For a tube—

$$Ioo = \frac{\pi}{32}\,(D^4 - d^4)$$

where D = outside diameter,

d = internal diameter.

Therefore

$$\text{torque} = \frac{f}{\frac{D}{2}}\,\frac{\pi}{32}\,(D^4 - d^4)$$

$$= \underline{\frac{\pi}{16}\,f\,\frac{(D^4 - d^4)}{D}}$$

EXAMPLE. The control lever on an aileron spar is 9 in. from centre line of control cable to centre of spar, and the load in the cable is 400 lb.

If the spar is a steel tube of 2 in. diameter and 0·08 in. thick, find the torsional stress in the spar.

$$T = PR = \frac{\pi}{16}\,f\,\frac{(D^4 - d^4)}{D}$$

$$f = \frac{PR \times 16 \times D}{\pi(D^4 - d^4)}$$

$$= \frac{400 \times 9 \times 16 \times 2}{\pi(2^4 - 1{\cdot}84^4)}$$

$$\doteq \frac{400 \times 9 \times 16 \times 2}{\pi \times 4{\cdot}53}$$

$$= \underline{8100}\text{ lb./sq. in.}$$

CHAPTER VI

THE STRUCTURE

The Main Planes

THERE are very few wings of the girder construction, shown in Fig. 49, on modern aircraft, but a great many are still in use.

The main members are the front and rear spars *A* and *B*, which are beams carrying distributed and end loads. If, as in the example, it is the wing of a biplane, attached to the spars are the interplane struts, flying and landing wires or struts, thus forming the "vertical" frames.

In the plane of the wing a complete frame is formed by means of the two spars together with the drag bracing *D* and the drag struts *C*. The struts run from front to rear and may either take the form of a metal tube, called a Drag Strut, as in the figure, or be a stiff member shaped to the form of the aerofoil, called a Compression Rib. This latter will be of the form of an ordinary rib specially strengthened between the spars to resist compression.

An advantage of compression ribs over drag struts is that they are able to support the spars against twisting.

The leading and trailing edges *E* and *F* are usually of tubular form; they support and keep the correct shape of the skin.

End ribs *G* and *H* must be specially strengthened in the horizontal plane to resist the pull of the fabric.

Where the wing is cut away for the aileron a special "False Spar" *L* is put to keep the shape of the wing at this point. This does not carry the aileron hinges. The aileron is carried by specially strengthened ribs *K*.

The fabric or skin is attached to the ribs by means of string stitching.

The function of the ribs is to keep the shape of the wing and to receive the air loads from the skin and transfer them to the spars.

They are beams carrying varying distributed loads from the air reaction and end loads due to the pull of the fabric.

In order to take the large loads between the front spar and leading edge, and to keep the shape at the most important part of the aerofoil, extra short ribs, called Nose Ribs (Fig. 49 (*M*)), are fitted at this part of the wing.

Most modern wings are of stressed-skin construction, and the design of these is so varied that only a few examples can be given here. It normally takes the form of a "box" beam. There are two main types: those with concentrated flanges and those with distributed flanges. Fig. 49 (*a*) shows an example of the concentrated flange type. The shear due to bending is taken by the spar webs, and if of the Wagner beam type we may expect them to form elastic instability waves under load. The shear due to torsion is taken by the top and bottom skin and the spar webs. Thus in flight, we may expect to see the skin form elastic instability waves. Stiff end ribs are used to give torsional stiffness, and intermediate ribs, whilst helping, mainly act as formers and give rigidity to the nose and tail portions. The end loans, due to bending, are mainly absorbed by the spar flanges. Fig. 49 (*b*) shows an

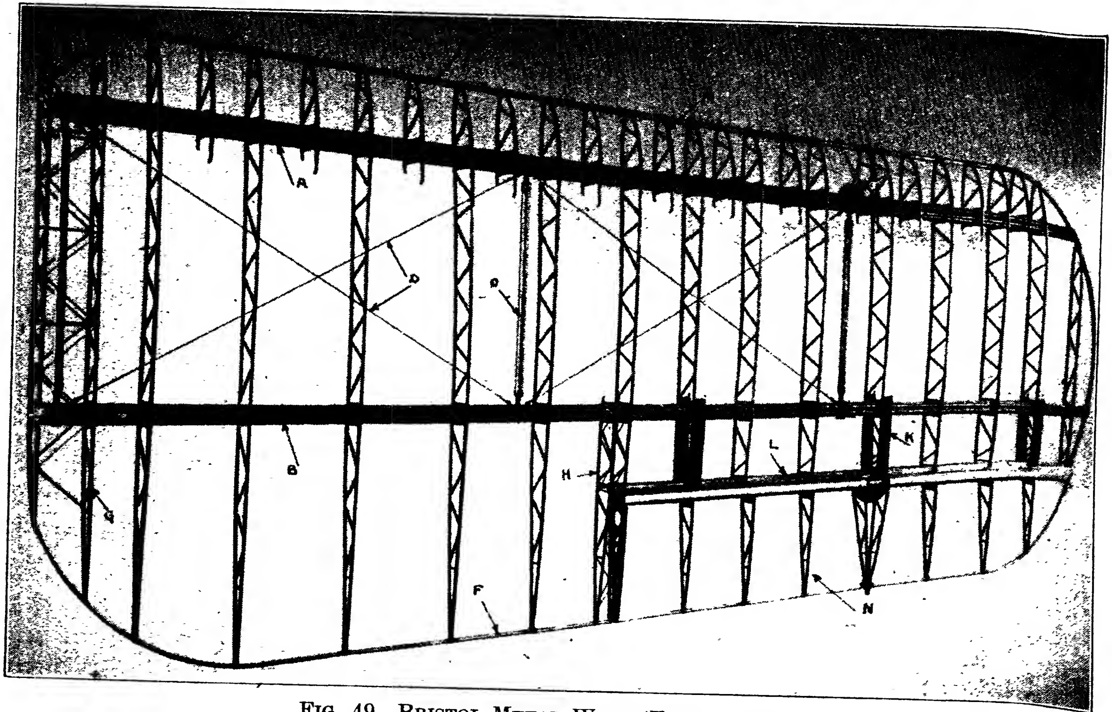


FIG. 49. BRISTOL METAL WING (EARLY TYPE)

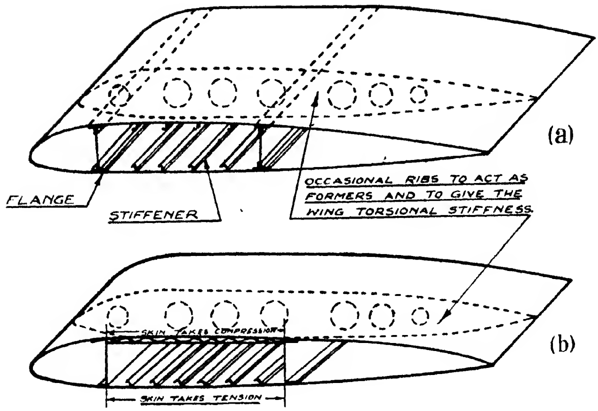


FIG. 49 (*a*) and (*b*).

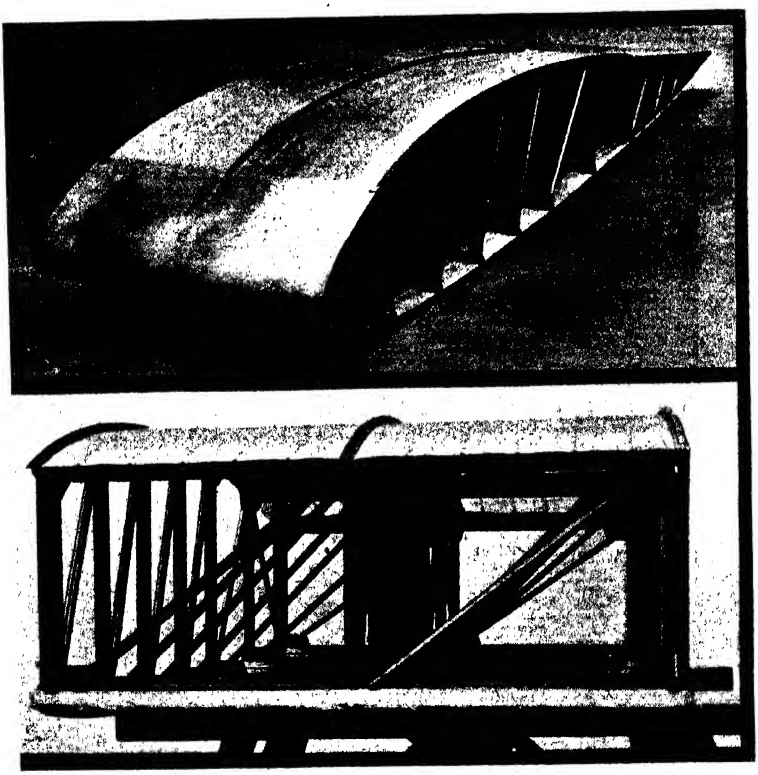


FIG. 49 (*c*). BRISTOL METAL-COVERED CANTILEVER WING STRUCTURE

example of the distributed flange type. The difference from the concentrated flange type is that the end loads are taken by the reinforced skin.

The early Bristol type given in Fig. 49 (*c*) may either be said to be an example of a multi-spar wing or the wing as a whole can be considered as a single spar. As will be seen, the structure is made up of rolled steel *N* girders, braced in the horizontal plane by the duralumin skin. The corrugated booms are laminated, the number of laminations increasing with the end loads towards the wing root.

The skin increases the strength of the booms, and acts as bracing to take shear and torsion.

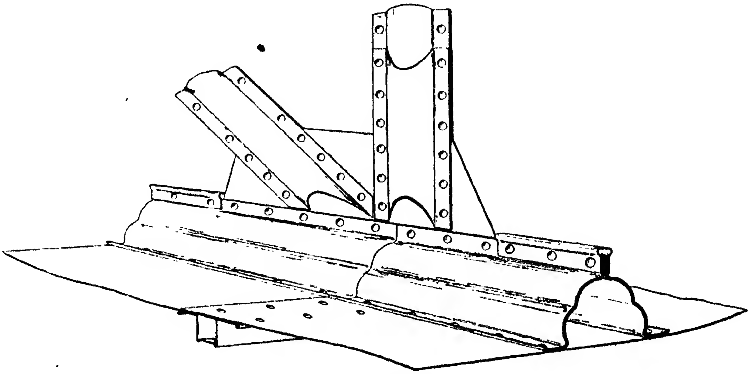


FIG. 49 (*d*)

Fig. 49 (*d*) shows the structure and the simplicity of the joints in greater detail.

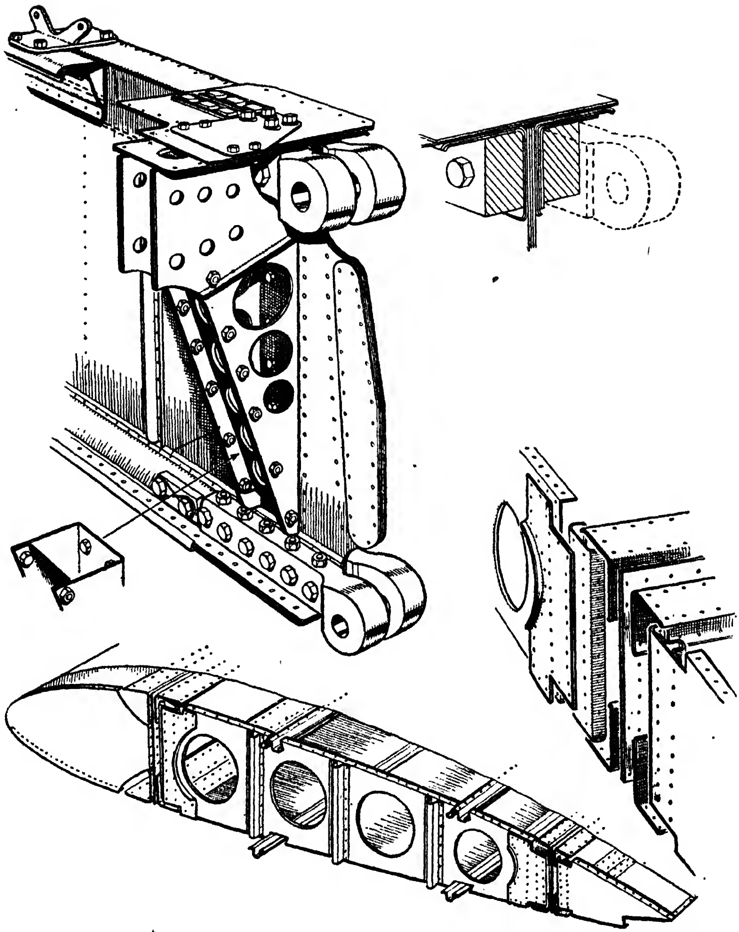
Fig. 50 shows details of the *Blenheim* wing construction, an example of concentrated flanges. The two spars have steel flanges and Alclad webs and are the Wagner type of beam. The skin, ribs, and stiffeners are also of Alclad. The root end fitting looks very solid, but it has to be able to take about 60 tons thrust in the pins. The ribs being stiff in torsion and the spars in flexure prevent any liability to flutter.

The *Ensign* wing construction is shown in Fig. 50 (*a*). The monospar is of light alloy, its corrugated flanges giving it a high resistance to compression and bending, while the small internal bracing tubes make it stiff in torsion. This is an example of distributed flanges.

The front and rear portions of the ribs are attached to the spar, which itself forms the centre portion. The covering is of light alloy, except aft of the spar, where fabric is employed.

An advantage of having it metal-covered in front and fabric-covered towards the rear is that it helps to mass-balance the wing. The necessity for getting the wing weight forward will become more important as higher speeds are obtained, giving increased tendency to flutter.

It may be noted here that metal-covered controls tend to bring their mass aft, making it more difficult to mass-balance them. This is one reason why many metal-covered aeroplanes still use fabric-covered controls.



(*From "Aircraft Production"*)

FIG. 50. STRUCTURAL DETAILS OF THE "BLENHEIM"

Fig. 50 (*b*) shows the *Battle* wing. It is constructed mainly of duralumin. Most of the stringers are extrusions, and the spar flanges at the root are built up of three steel plates, on each side of which are channel-section extruded duralumin members, and then four more steel side plates. Near the root the flanges are braced together, but this gives place to duralumin webs towards the tip. From the root outwards the number of plates and flanges are decreased and the inner flange of the extruded channel is cut off. The ribs are pressed from duralumin sheet.

FLEXIBILITY PROBLEMS*

In considering flexibility, the following terms will be used—

Semi-rigid Wing. A wing that distorts so that straight lines in the direction of the span remain straight, flexure occurring only at the root. A conception used for simplification.

Flexural Centre. The point in the section of a wing where, if a force be applied, flexure occurs without twist.

Centre of Independence. A point in an aerofoil section, at about ¼

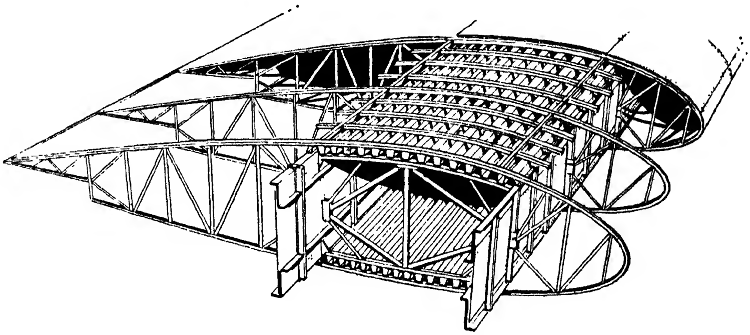


FIG. 50 (*a*) (*By courtesy of "Flight"*)

chord from leading edge, where the moment of the aerodynamic forces is constant with angle of attack.

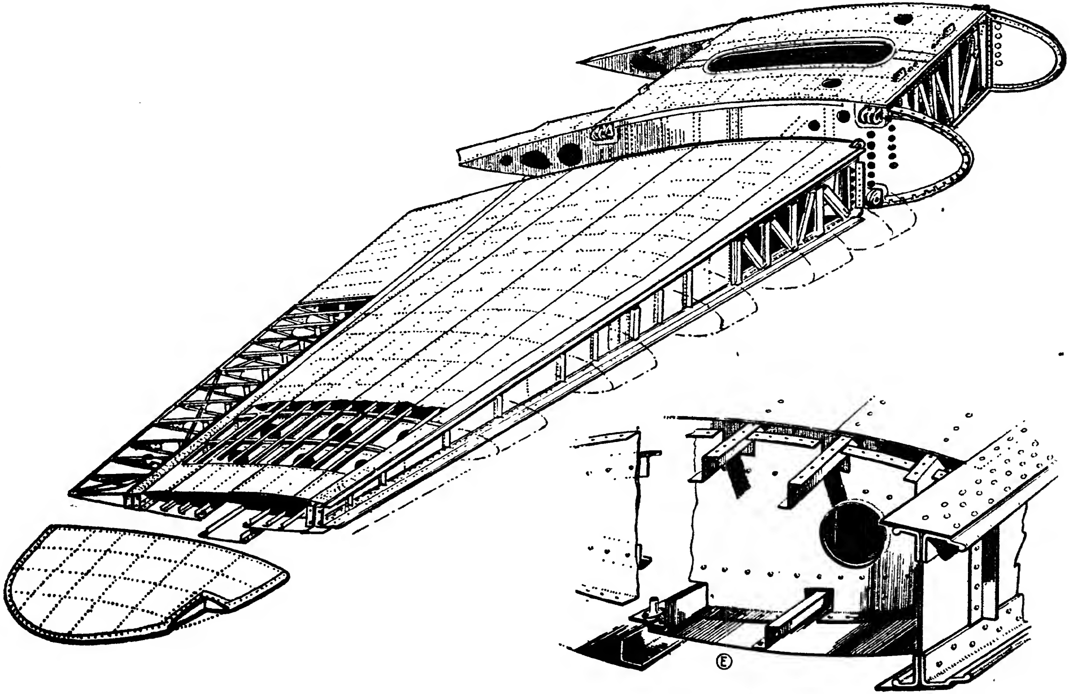
Consider a portion of a wing supported by a rod (Fig. 51). A is the flexural centre. It is obvious that a force applied at any other point will twist the wing. Let Z_1 be the component of the aerodynamic reactions normal to the chord, and Z_2 the normal component at an increased angle of attack. Then, if $Z_1a_1 = Z_2a_2$, A is also the centre of independence. The supporting rod will apply a resisting torque, and any change in angle of attack (other things being equal) will not change this torque, and therefore will not further twist the wing.

Wing Twist

By normal methods of stress analysis it is assumed that the configuration of the structure is the same in the stressed and unstressed conditions. Actually, due to straining, this is not true, but the changes are normally so slight as to be negligible. A wing structure is comparatively flexible, and, although this flexibility would not normally affect the stress analysis, the change in aerodynamic loads from this cause would. If we plot a graph connecting wing twist and applied torque, we will get a curve of the form shown by ABC in Fig. 51 (*a*), B being the elastic limit.

Consider a section of an unstrained semi-rigid wing in air flow of velocity V and angle of attack α. Let aerodynamic moment about the

* For fuller explanation, see *Journal of Royal Aeronautical Society*, Sept., 1933, and Feb., 1934.



(From "Aircraft Production")

FIG. 50 (b). THE CENTRE SECTION, MAIN WING, AND WING TIP ARE MADE AS THREE SEPARATE UNITS

(E) is a detail of the front main spar, showing the attachment of ribs and "Z" stringers

flexural centre (= applied torque) be represented by M on the graph. Let β be the angle of attack at which there is no aerodynamic moment about the flexural centre. We may assume that aerodynamic couple

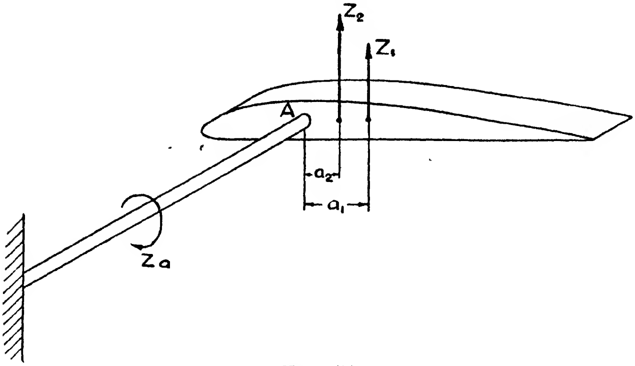


FIG. 51

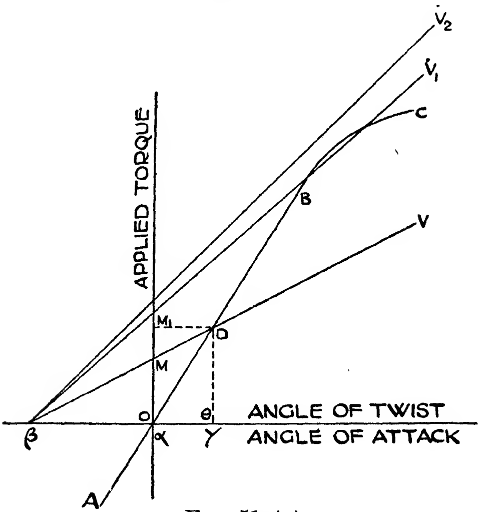


FIG. 51 (*a*)

and angle of twist give a linear law, then MDV is the graph connecting them.

As we assumed the wing was unstrained at angle of attack a, there will be no resisting torque, and the wing will twist until the elastic resisting torque is equal to the applied aerodynamic couple. Point D shows where equilibrium is obtained, i.e. the section will twist through angle θ to angle of attack γ. The above is illustrated diagrammatically in Fig. 51 (*b*).

Increase the speed, and the aerodynamic couples will be increased for a given angle of attack. If the speed is such that the aerodynamic couple curve V_1 cuts ABC above B, a permanent set will result. Increase the speed still further to V_2, where the couple curves never cut, and the wing will twist until failure occurs. The lowest speed at which this occurs is called the *divergence speed.* Such a speed on a very flexible wing may be reached in high-speed flight.

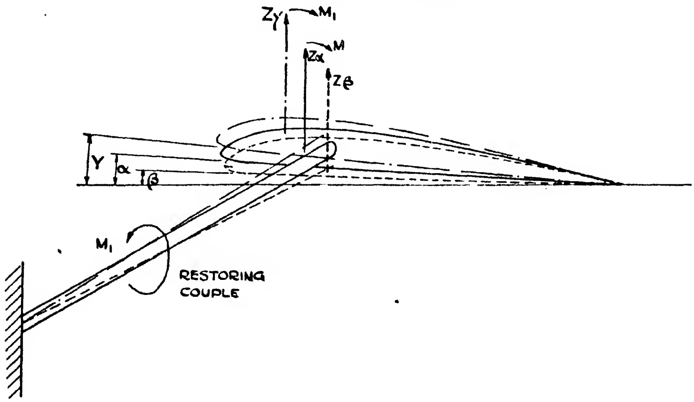


FIG. 51 (*b*)

Effect on Lateral Control

When the ailerons are moved, a moment is produced about the flexural axis which twists the wing in such a way as to reduce the aerodynamic forces brought into action, e.g. if aileron moves up, aerodynamic force at the rear of the wing is downwards, giving a nose-up moment about the flexural axis. Thus the angle of attack is increased, so producing an increased upward reaction, which may, if the wing is not sufficiently stiff in torsion, produce an actual reversal of control. The speed at which this occurs is called the *reversal speed.*

FLUTTER

Wing Disturbed in Flexure

If a wing whose centre of mass does not coincide with its flexural centre is displaced flexurally without twist in still air and released, twist will occur due to the inertia forces, e.g. if the centre of mass is behind the flexural centre and the wing is accelerating downwards, the inertia acting at the centre of mass will twist the wing nose-down. The resultant flexural and torsional oscillations will be damped out as the wing does work against the air. If, however, the wing is in an air stream, the change in angle of attack due to twist and speed of flexure will bring into play aerodynamic forces which may increase the oscillations.

Consider a wing in an air stream moving downwards in flexure. The angle of attack will be increased, due to the downward velocity of the wing, thus giving an increased aerodynamic reaction, which will exert

an increased twisting moment about the centre of flexure unless the centre of independence and flexural centre coincide. This is reversed when the flexural velocity is upwards, so giving rise to a torsional oscillation. The inertia forces due to the torsional oscillation may give rise to a flexural movement, thus renewing the initial condition. So a cycle is set up involving a flow of energy from the air to the wing.

If the air speed is sufficient for the increased reactions to overcome the damping effect of the air, the oscillations will increase in amplitude until fracture occurs. This phenomenon is called *flutter.*

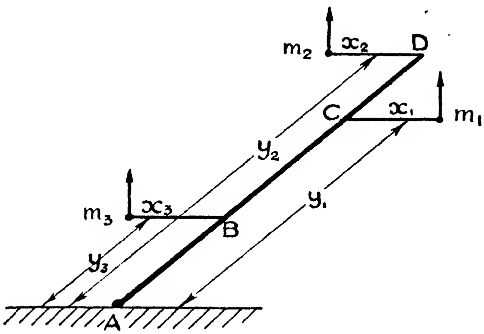


FIG. 52

Wing Disturbed in Torsion

Now consider a wing initially disturbed in torsion. The twist changes the angle of attack, giving rise to an increased aerodynamic reaction below the stall, if the twist is nose-up. This in turn tends to flex the wing upwards, or downwards when the twist is nose-down, giving rise to flexural oscillations. Inertia will, if the centre of mass does not coincide with the flexural centre, twist the wing as previously described, thus completing the cycle.

Prevention of Flutter

We have seen how flutter is brought about by cycles: Flexure—Torsion—Flexure, or Torsion—Flexure—Torsion. If we can destroy a link in each of these cycles, we can prevent flutter.

We may prevent flexural motion giving rise to torsional motion due to inertia by distributing the mass of the structure so that at any section the flexural centre and centre of mass coincide. This, however, is not practical, but we can get the same effect on a semi-rigid wing if $\Sigma mxy = 0$. That is to say, if the whole mass of the wing be split up into a number of very small masses, and each small mass m multiplied by its distance x behind the flexural axis and its distance y from the wing root, the sum of the products mxy of all the small masses of the wing must be zero. This is called *mass balance.*

This is represented diagrammatically in Fig. 52. Consider the rod $ABCD$ hinged at A and accelerating downwards. Mass m_1 will exert an inertia force upwards m_1a_1, where a_1 is acceleration at C. This gives an anti-clockwise torque $m_1a_1x_1$. Now add mass m_2. As $ABCD$ flexes

only at A (semi-rigid wing), acceleration of $m_2 = a_1y_2/y_1$, and its clockwise torque $= m_2x_2a_1 \cdot y_2/y_1$.

In the same way torque of $m_3 = m_3x_3a_1 \cdot y_3/y_1$.

To prevent twist—

$$m_1a_1x_1 + m_2x_2a_1\, y_2/y_1 + m_3x_3a_1\, y_3/y_1 = 0$$

i.e. $$a_1/y_1\,(m_1x_1y_1 + m_2x_2y_2 + m_3x_3y_3) = 0$$

$$\text{or } \Sigma\, mxy = 0$$

(Note x_2 and x_3 are negative.)

It will be seen that a mass near the tip will have a greater effect than one near the root where y is small.

Flutter would be absolutely prevented if now we could stop flexural motion giving rise to torsional motion, due to aerodynamic reactions.

This means designing a wing with the flexural centre coinciding with the centre of independence, which it is difficult to do, so it is left to reduce the effect of the aerodynamic reaction by increasing the wing stiffness, and thus increasing the speed at which flutter will occur.

Effect of Control Surfaces

So far we have neglected the effect of ailerons, but they will have no effect if they behave as though locked to the wing. This may be brought about by irreversible control, which at the moment is not practicable.

A partial solution is given by—

(1) Arranging the mass so that Σmxy is slightly negative (x is distance behind hinge line for controls). This negative value of Σmxy causes the aileron to be deflected when the wing is accelerating in flexure downwards. This gives a tendency for increased lift, which tends to counteract a reduction of lift caused by the inertia of the wing twisting it nose down.

(2) Making the moment of inertia about the hinge axis as small as possible.

(3) Elimination of slack in the cables.

Flutter may also occur in tail units, and the above considerations also apply there.

Modern Tendencies

Wing-aileron flutter has been controlled by mass balance, etc., since about 1930, but now there is the choice between increased wing stiffness or wing mass-balance to overcome the increased tendency to wing flutter and aileron reversal, due to modern high speeds and wing loadings.

Increased wing stiffness would mean either a stiffer type of structure such as geodetic, increased skin thickness, or stronger ribs, resulting in increased weight, or reduced aspect ratio and/or increased depth of wing, giving less aerodynamic efficiency. For a stressed skin wing the speed at which flutter occurs roughly varies inversely with aspect ratio, and directly as the thickness-chord ratio.

Mass balancing, by disposing the wing structure weight forward, or by the rearrangement of wing engine position, may be more efficient, though some increased torsional stiffness may be required to prevent aileron reversal. Taper helps to increase the reversal speed.

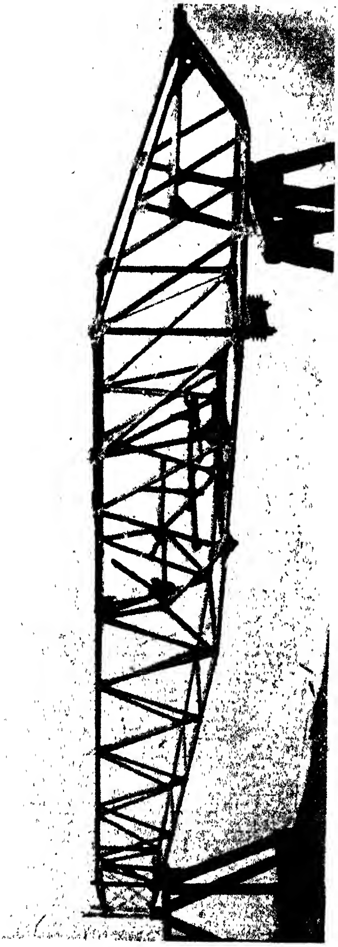


Fig. 53 (*a*)

Fuselage

The most common type of fuselage used in the past was a built-up girder. Four longitudinal members, called Longerons, are braced by a system of panels made up of vertical and horizontal struts braced

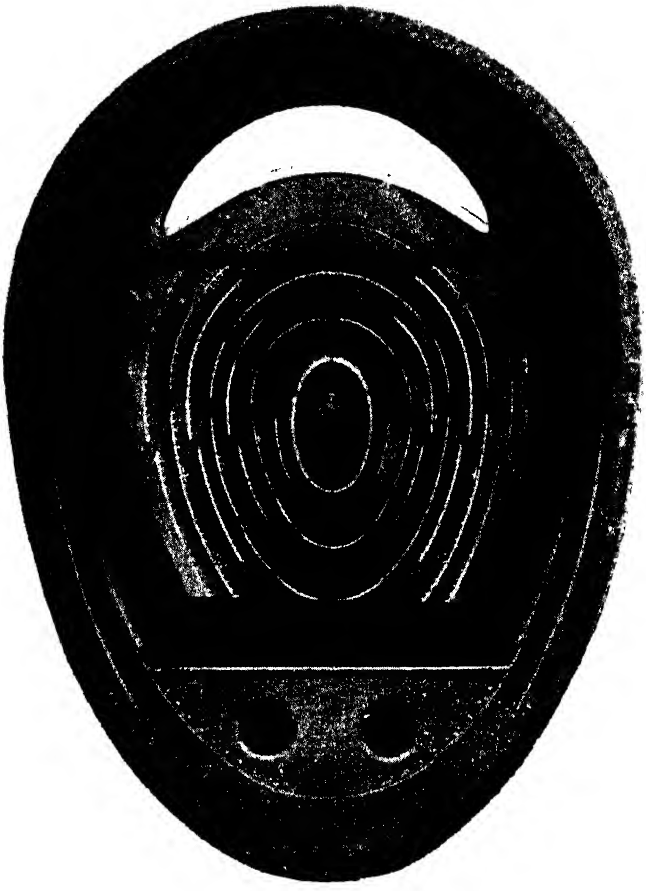
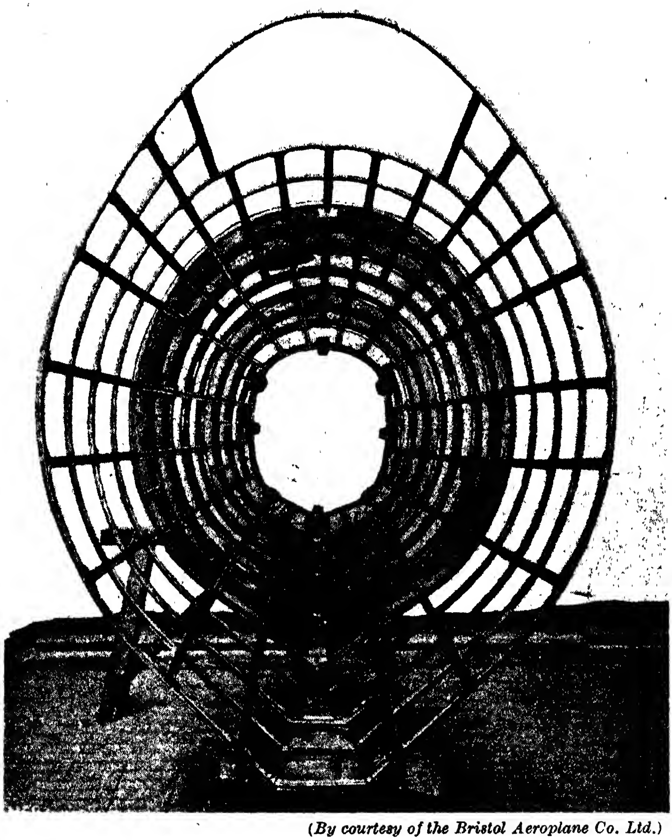


Fig. 53 (*b*)

together by either cross-wires or a diagonal strut, the whole forming a beam supported by the wings in flight and by the chassis and tail-skid in landing. An example, by Hawkers, is shown in Fig. 53 (*a*).

In order to reduce head resistance, the fuselage is made as near a streamline shape as is practicable. This is done by attaching a light structure, called a Fairing, over the main structure. This fairing is solely to give shape, and does not increase the strength.

A type of fuselage which was first tried for stressed skin construction was the Monocoque. It consists of a thin cylindrical shell stiffened with transverse formers. There are no bracing wires or longerons, so that the skin must withstand all the loads, which are bending and



(*By courtesy of the Bristol Aeroplane Co. Ltd.*)

Fig. 54

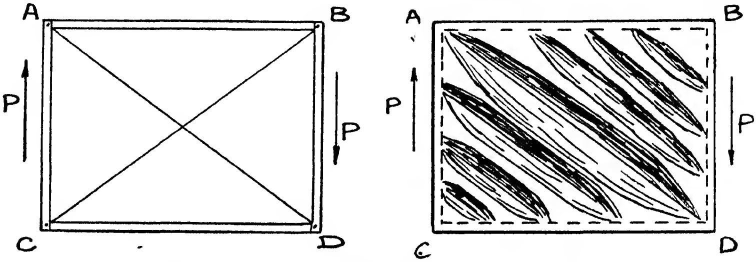
shear. Light longitudinal members are riveted to the skin between the formers in order to stiffen the skin and so increase the stress at which elastic instability occurs.

Advantages of monocoque construction are: the whole of the structure being in the skin a very roomy fuselage is obtained, also a good streamlined shape and smooth surface are obtained, thus reducing drag.

The utilization of the skin for taking the load would at first appear to result in a big reduction in weight, and this would be so if the skin

developed its full compressive stress. Considering elastic instability, we have seen that, in order to develop the full stress of the material, the diameter-thickness ratio must not exceed 100. Thus a 2 ft. diameter fuselage would have to have a ¼ in. thick skin. It follows, therefore, that with thin skin failure takes place, due to elastic instability at a very low stress. This stress depends mostly on the thickness of the skin and the number of longitudinal stiffeners, and is usually in the order of 5 tons/sq. in.

As the strength is independent of the strength of material used, we



(*a*) FIG. 55 (*b*)

must choose a material of low density, to enable us to use sufficient thickness of wall to give stability under all required loads, and at the same time keep the structure light. Thus light alloy or plywood is preferable to steel.

De Havilland use the very light balsa wood, sandwiched between plywood to give thickness, so giving much extra strength with little increase in weight.

A good example of monocoque construction is the Kellner-Bechereau fuselage shown in Fig. 53 (*b*).

The type of construction in greatest use to-day is the semi-monocoque. It enables a thinner skin to be used efficiently. An example due to the Bristol Aeroplane Co. is shown in Fig. 54. It will be seen that it consists of a framework of longitudinal and transverse stringers, to which is riveted the thin skin.

The stringers, helped by the skin in their vicinity, support the compressive and tensile loads, which occur due to bending. Shear loads and torsion will tend to distort the panels, and as these are braced by the skin, they will put tensile and compressive stresses in the skin diagonally across the panel. Consider a wire braced panel, Fig. 55 (*a*). Under the shear force P the wire AD will be in tension, whilst CB will slacken off, being unable to resist compression; it will in fact be elastically unstable. The thin skin braced panel, Fig. 55 (*b*), will similarly be in tension across AD, and in compression across CB; the skin across CB being elastically unstable. The skin being riveted to the frame cannot bow in one curve as did the wire, but will buckle into the usual waves typical of elastic instability, at very low values of P. The load is then taken entirely by tension across AD. See also Fig. 42.

The stress should not exceed that at which the waves become permanent, although the structure may be still safe under that condition.

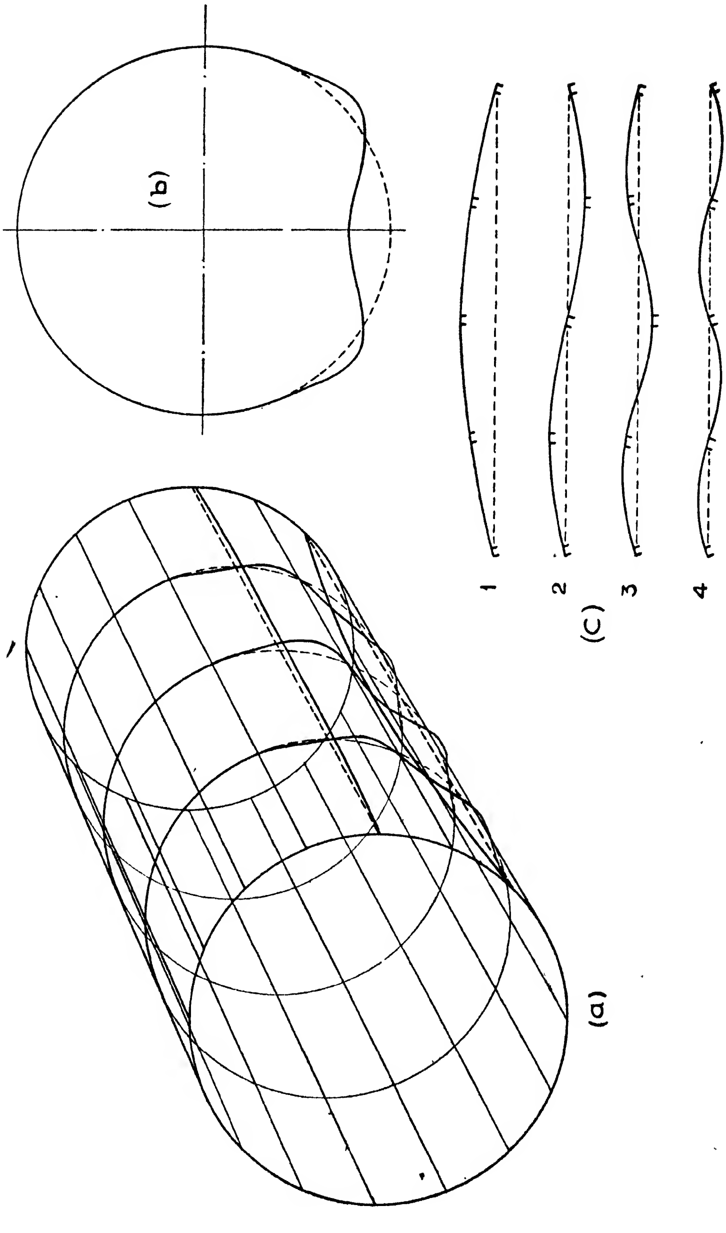


FIG. 56

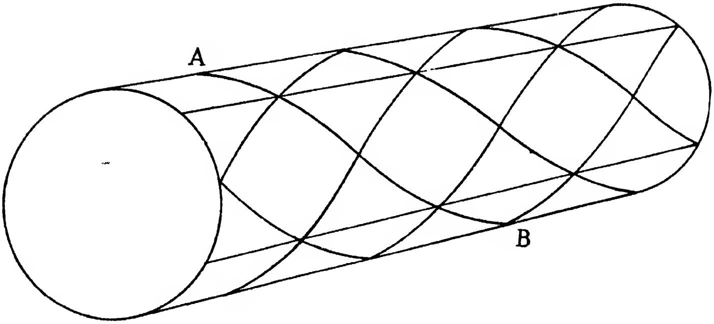


FIG. 57

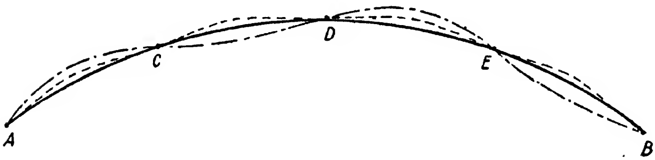


FIG. 57 (*a*)

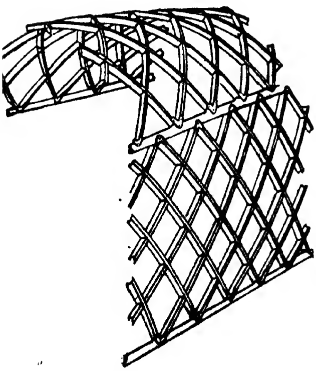


FIG. 57 (*b*)

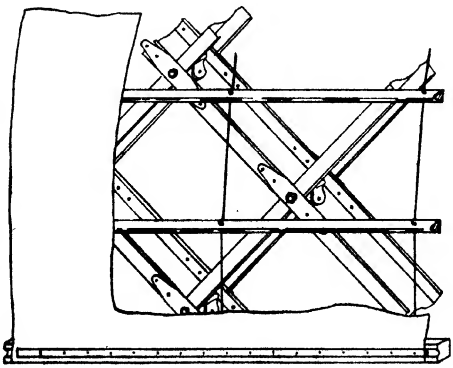


FIG. 57 (*c*)

Too big a wave may have a detrimental effect on the stringers, helping to buckle them under compression.

If the transverse stringers or rings are weak, the structure will deform on the compression side as shown in Fig. 56 (*a*), the rings deforming as shown in Fig. 56 (*b*) and the longitudinal stringers as Fig. 56 (*c*) (1).

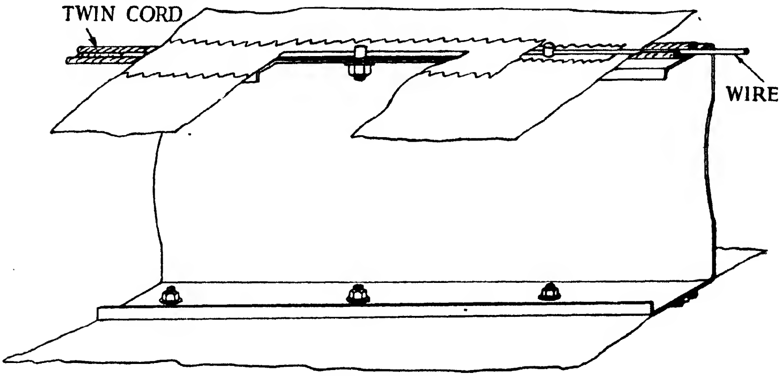


FIG. 57 (*d*)

If the rings are stiffened, the stringers will be unable to deform in a single bow, but will deform as in Fig. 56 (*c*) (2), (3), or (4), depending on the stiffness of the rings. Thus it will be seen that by stiffening the rings we shorten the effective length of the stringers as struts in compression, and so decrease their elastic instability.

Geodetic Construction

This type of construction is used by Vickers on their fuselages and wings. It gives great torsional stiffness, allowing a high aspect ratio to be used on the wings, without the tendency to flutter or aileron reversal which would occur on other types of construction at high aspect ratios.

Take a cardboard cylinder (Fig. 57) and stretch a thread round it, so that between any two points A and B it has taken the shortest distance. Repeat with other threads in the same direction, and again in the reverse direction. Now replace the threads by wires, and add four longitudinal wires. Solder all points where wires cross, and remove the cardboard. The result is a model of a geodetic fuselage.

Apply a torque, or shearing force, and the wires in one direction will be in tension, while those crossing them will be in compression. Consider the length of wire AB to be in compression. As it is initially bowed, it would by itself take little load, but as it is crossed by three wires carrying an equal tensile load, the tendency for AB to bow further at the joints is counteracted by an equal tendency of the tensile wires to straighten. Thus the points at which the geodesics cross are fixed.

Consider the geodesic A–B initially bowed, but for simplification we will consider it all in one plane, as Fig. 57 (*a*). Let points C, D and E be the fixed points where the other geodesics cross A–B. If A–B was a straight member it would deflect under compression, as shown by

the chain-dotted line, but as it is already bowed it can only deflect in its initial direction (analogous to a knee joint), as shown by the dotted lines. That is each portion between joints acts as a rigidly fixed strut, or a pinjointed strut of half the length of the unsupported portion. This means that the shortening of A–B will be comparatively small, making the structure stiff in torsion in relation to the weight of material used.

Fig. 57 (*b*) shows a top and side of a portion of the *Wellington* fuselage. The geodesics are broken at the longerons for ease of construction, but as the unsupported lengths of the longerons are very short, the resultant bending moment in them is small. The longerons take the tension and compression due to bending, and the geodesics shear and torsion, while they also support the longerons against buckling. Fig. 57 (*c*) shows the

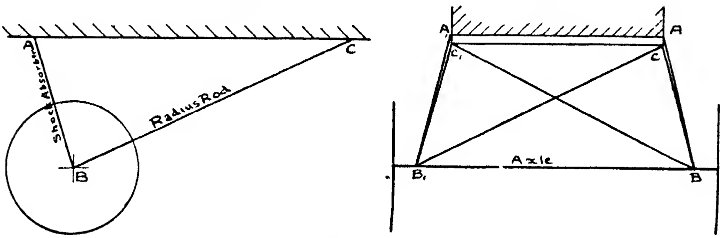


FIG. 58

form of the geodesics, and the way they are fixed to each other at the cross-over joints. This figure also shows how the fabric is attached and prevented from chafing against the structure. Fig. 57 (*d*) illustrates the attachment of the fabric to the wings.

The Undercarriage

The undercarriage in conjunction with the tail skid has to absorb and dissipate the shock (i.e. kinetic energy) on landing and form a carriage to enable the aeroplane to taxy easily over the ground.

Although undercarriages vary in arrangement, essentially they are comprised of wheels, axle, shock absorbers, and bracing. The most simple arrangement is shown diagrammatically in Fig. 58.

The cross bracing wires CB_1 and C_1B are in the plane of the radius rods BC and B_1C_1, which must be free to rotate about C and C_1 respectively, as the shock absorbers AB and A_1B_1 vary in length.

The landing loads on the machine will decrease as the give of the shock absorbers increases.

There are several types of shock absorbers. A type which was in common use during the 1914-1918 war consists of a vee-strut connected to the axle by means of shock absorber cord bound round with an initial tension. On landing the cord will stretch, thus reducing the shock.

Another and later method uses coiled steel springs. As, however, these will absorb the energy of the shock, but only dissipate it to a small extent, a vibration damper must be used. This usually consists of an oil dashpot, called an Oleo Leg, a diagrammatic sketch of which is shown in Fig. 59. It consists of two telescopic tubes A and B, the upper one attached to the body and the lower outer one to the axle. In flight the leg is extended, all the oil being contained in the bottom tube.

When the machine strikes the ground the oil passes through the small holes *C* into the upper tube. If the impact force is such that it exceeds a certain figure, the pressure of the oil will raise the spring-loaded valve *D*, thus providing an additional passage. By this means the maximum landing load is regulated by the valve spring used, unless the landing

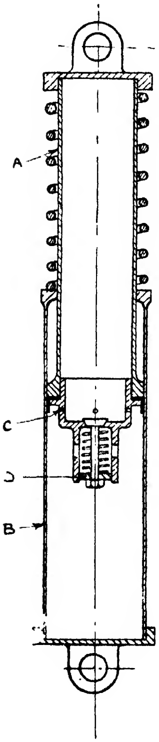


FIG. 59

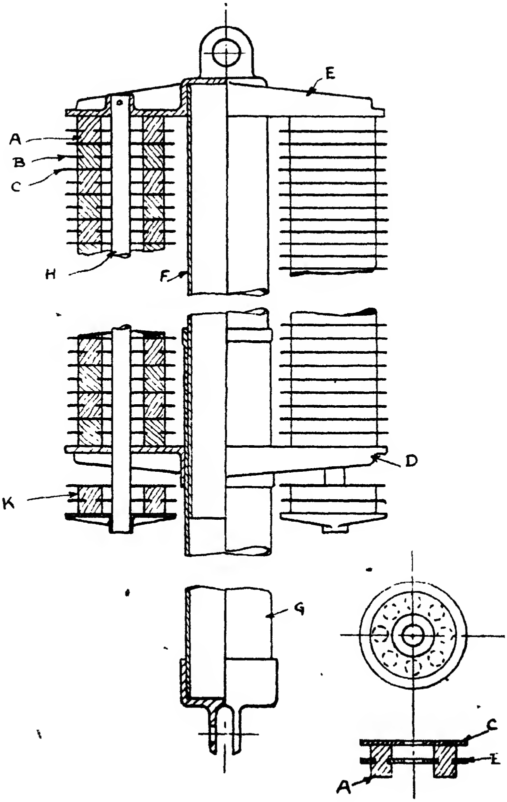


FIG. 60

is so bad that the whole of the travel is taken up. On the return stroke the valve will be closed and the oil must return through the holes *C*, thus preventing bounce. In addition to the oil gear, spiral springs are provided to carry the weight of the aeroplane when running along the ground, and to re-extend the leg. The energy absorbed by the spring is dissipated on the return stroke, by the oil having to again pass through holes *C*.

A modification of this type, which has the advantage of lightness, is the Oleo-pneumatic Leg. In this there is no spring and the chamber *A* (Fig. 59), which is airtight, contains air at an initial pressure sufficient to support the aeroplane when standing on the ground. As the oil is

forced through the valve, under any increased load, the air is further compressed, thus increasing the load required to compress the leg as it approaches its maximum travel. On any reduction of the load the air will force the oil back through the valve.

Another type makes use of rubber blocks in compression. A sketch of an undercarriage leg, showing the arrangements of the various components, is shown in Fig. 60.

The rubber *A* is moulded in one piece through the holes in the stabilizing plate *B*. To enable the rubbers to spread easily under load, a separate plate *C* is placed between each rubber.

As the load is applied it is transferred by the flanges *D* and *E* to the rubbers, which deflect and allow the tube *F* to slide in the tube *G*; at the same time the tube *H* will move freely through the flange *D*. The rubbers *K* act as a buffer on the return stroke.

Compression rubbers will dissipate the energy absorbed better than springs and tension rubber, and may therefore be used without a damping device.

An oleo leg is, however, often included to decrease the tendency of the machine to bounce.

The stress in the rubber should not exceed 250 lb. per square inch, or it will lose a portion of its elastic properties. Below this stress its life should be about two years.

In all undercarriage systems a large proportion of the shock is absorbed by the pneumatic tyres.

In a recent type of undercarriage the shock-absorbing device is incorporated in the wheel hub, and may be of the oleo-spring or oleo-pneumatic types. An advantage of this type of wheel is the low weight, and the reduction of drag, due to the simple mounting.

An example of a Dowty internally-sprung wheel for cantilever mounting is shown in Fig. 61 (*a*). The axle is fitted to the socket on the shock-absorbing unit, but the wheel obviously rotates about its centre, on the large diameter bearing.

Retractable Undercarriage

Largely due to the necessity of greater reduction of drag with increasing speeds, retractable undercarriages are becoming generally used to get rid of the high parasitic drag of this unit. At the same time the use of thick cantilever monoplane wings has provided an easy storage for the retracted unit.

The following are the main design requirements for efficient retraction—

1. Housing in wings or fuselage should be closed by doors or fairing after retraction.
2. The undercarriage must be raised and lowered with reasonable speed (about 30 secs.) and without undue exertion on the part of the pilot.
3. Position locks must be provided to hold the undercarriage in the landing position.
4. An indicator on the dashboard should show when the undercarriage is locked in the landing position, and when fully retracted.
5. An audible indicator should be arranged to sound, if the undercarriage is not locked for landing, when the throttle is less than one-third open.

6. Where power-operated mechanism is used an additional manually-operated gear should be provided, in case of failure.

Fig. 61 (*b*) shows a Dowty system of retraction. The folding strut is hinged at *A* and folded due to the extension of the hydraulic jacks *B*.

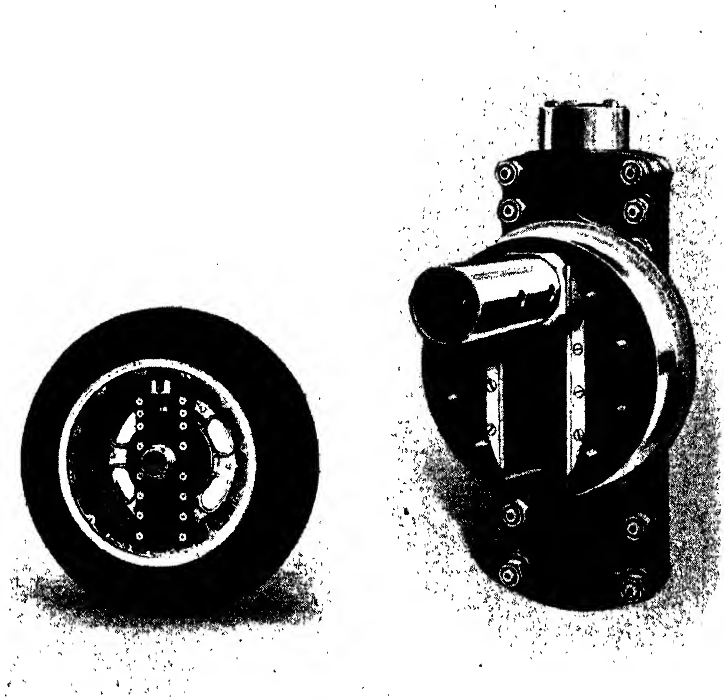


FIG. 61 (*a*)

When retracted the housing is faired by *D*. There is a radius rod, not shown, in the plane of the shock-absorbing leg *C*.

Hydraulic Systems

Retractable undercarriages, flaps, etc., are usually worked by means of hydraulic pressure. The property of a fluid to transmit pressure equally in all directions is what makes it so suitable for transmitting power from a single source to remote units, which would otherwise require complicated and heavy gearing.

The simplest hydraulic machine is the hydraulic jack, shown diagrammatically in Fig. 61 (*c*). A force f on the plunger P applies a pressure to the fluid equal to f/a, where a is the cross-sectional area of the plunger. This pressure is transmitted equally throughout the fluid,

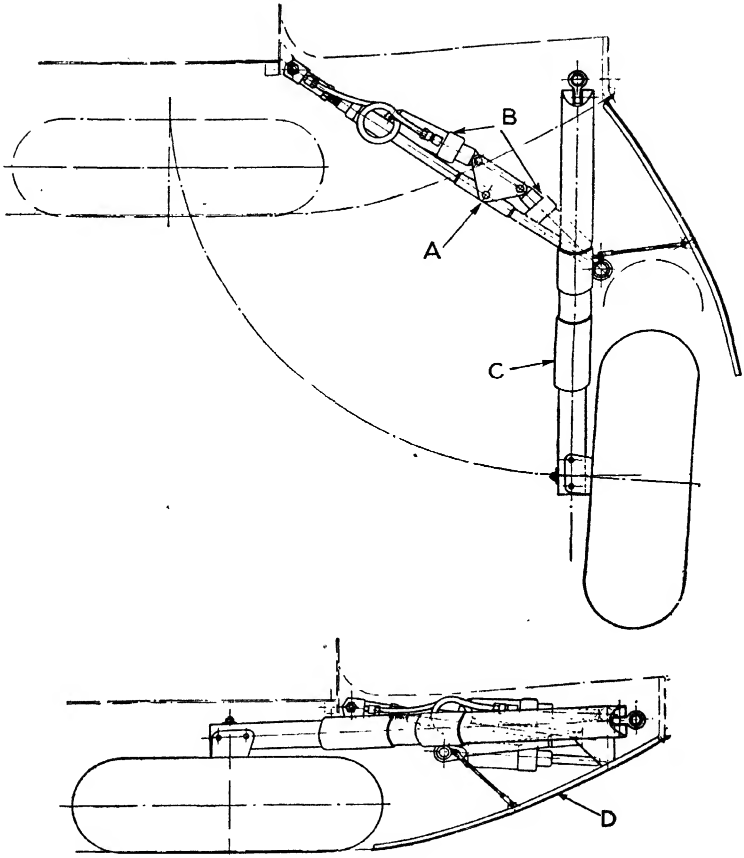


FIG. 61 (b)

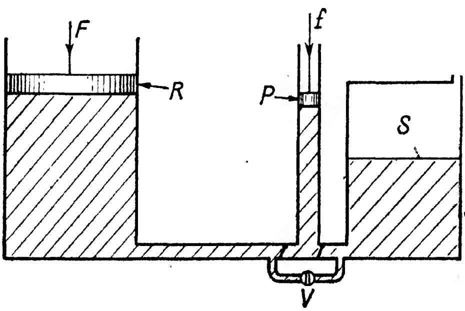


FIG. 61 (c)

so that the fluid exerts a pressure on the ram R equal to f/a. If A is the cross-sectional area of the ram, the lifting force F on the ram

$$= \text{pressure} \times \text{area}$$

$$= \frac{f}{a} A$$

Thus for a given effort the load lifted depends on the relative areas of the ram and plunger.

One stroke of the plunger would not lift the ram very far, so a reservoir S is provided, and the plunger pumps fluid from the reservoir

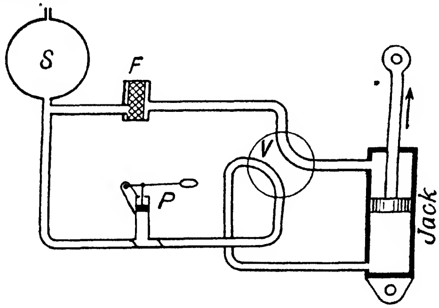


FIG. 61 (*d*)

and delivers it under pressure to the ram. To lower the ram the valve V is opened, permitting the fluid to return to the reservoir.

The essential difference between the jack used for aircraft and the lifting jack is that the former must be moved by hydraulic pressure in both directions. The minimum essentials for working such a jack are shown in Fig. 61 (*d*).

With the control valve V in the position shown, the hand pump P drives fluid under pressure into the lower cylinder of the jack, forcing the ram upwards. The fluid from the upper cylinder passes through the low pressure pipe line and filter to supply fluid to the pump. As the lower cylinder has a greater capacity than the upper cylinder, due to the ram-rod, extra fluid is required during the upward stroke. This is supplied from the reservoir S.

For the downward stroke the control valve V is turned through 90°.

The fluid used in aircraft systems is a mixture of alcohol and castor oil, which has the advantage of lubricating the system, whilst it does not freeze nor vaporize at the temperatures met with in practice.

A complete system for working the undercarriage and flap jacks is shown in Fig. 61 (*e*). Only one of each pair of jacks is shown for simplicity. The engine driven pump delivers fluid continuously, while the engine is running, to the automatic cut-out valve which normally allows the fluid to flow back to the reservoir, via the filter, at low pressure. Under this condition the pump is idling and little power is required to run it. A diagram of the cut-out valve under this condition is shown in Fig. 61 (*f*).

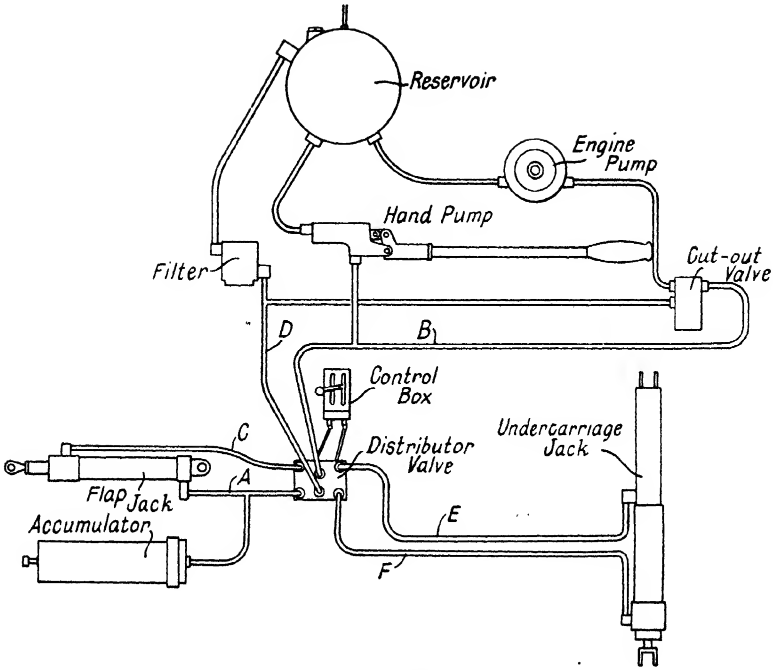


FIG. 61 (e)

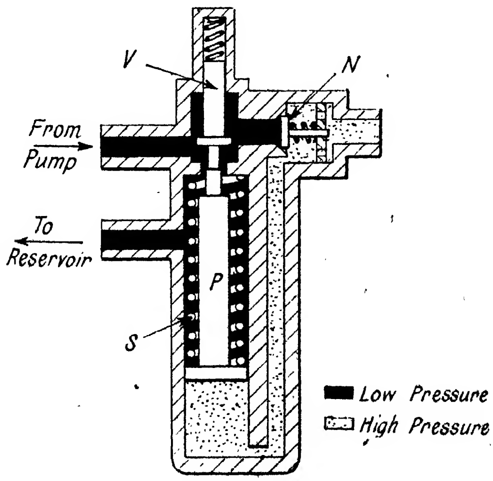


FIG. 61 (f)

To work the undercarriage, jack pipe B, containing fluid at high pressure, is connected to pipe E, and pipe F is connected to the return pipe D by means of the distributor valve, worked from the lever control box in the cockpit. The ram starts to move, causing a drop of pressure in the pipe B, which allows the spring S of the cut-out valve to lower the piston P, thereby allowing the valve V to seat and close the line from the pump to the reservoir. The pressure of the fluid from the pump will rise, open the non-return valve N, and fluid will be delivered under pressure to pipe line B, as shown in Fig. 61 (g). When the undercarriage jack ram has reached the limit of its travel pressure will increase in the pipe line B and on the base of the cut-out valve piston P, forcing it up against the spring S. The valve V is then lifted by P, allowing fluid to escape to the reservoir, whilst the non-return valve N closes, keeping up the pressure in the pipe B, but allowing the pump to idle.

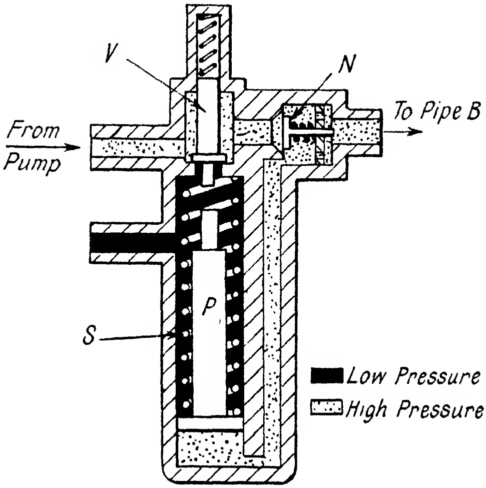


FIG. 61 (g)

For the return stroke of the ram the distributor valve is moved to connect pipe lines F to B and E to D, the cut-out valve functioning as before.

After each operation the control lever is returned to neutral. This locks the fluid in the pipe lines on the jack side of the distributor valve. To prevent undue pressure occurring in these pipe lines due to thermal expansion of the fluid resulting from change of temperature, an automatic relief valve is incorporated in the distributor unit.

When the undercarriage falls by its own weight, for part of its travel fluid will be forced out of the lower end of the jack, but the pump will not supply fluid fast enough to keep full the lower cylinder, with the result that the fluid will vaporize, causing a partial vacuum above the ram. To avoid the time lag required to fill the upper cylinder before further lowering of the undercarriage can take place, a short circuiting valve is often used to connect line E to F for the period during which the undercarriage falls under gravity, so that the upper cylinder is kept filled by the fluid forced from the lower cylinder.

The hand pump is provided as a standby in the event of the failure of the engine, and so of the engine pump. The non-return valve incorporated in the cut-out valve prevents the fluid leaking back to the

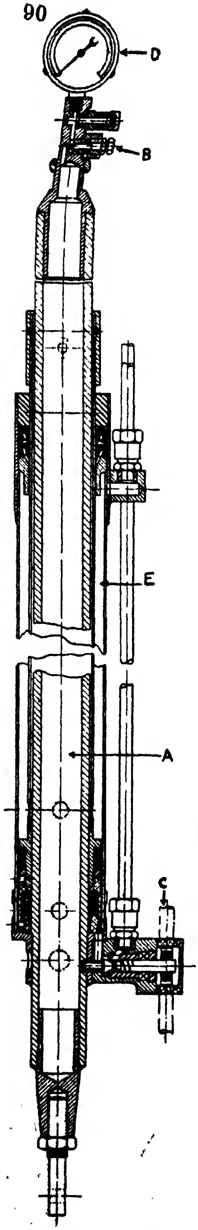


FIG. 61 (*h*)

reservoir via the engine pump, when the hand pump is in use. Systems not using an automatic cut-out valve *must fit a relief valve in the same* position *in the circuit, so that when the rams have reached the limit of their travel* excessive pressures will not be built up. A non-return valve must also be placed between the hand and engine pump. Idling can be obtained by connecting pipe lines *B* and *D* by the distributor valve when the control lever is in the neutral position.

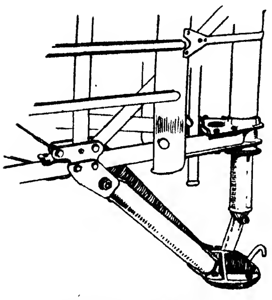
To lower the flaps pipe *B* is connected to pipe *A* and *D* to *C*, to return the flaps *C* and *B* and *A* and *D* are connected. In order that excessive loads will not be put upon the wings in the event of the aircraft being dived with the flaps down, either a relief valve must be fitted to connect *A* to *C* when the pressure rises, due to the load on the flaps, above a certain limit, or a hydraulic accumulator must be fitted to pipe line *A*. The hydraulic accumulator consists of a cylinder open at one end to pipe *A*, and containing compressed air at the other end; the two ends being separated by a floating piston. Any excess force on the flaps increases the pressure in pipe *A*, and on the base of the floating piston sufficient to raise it against the air pressure. As a result fluid flows from the flap jack into the accumulator, raising the flaps and decreasing the force on them. When the load on the flaps is reduced the compressed air forces the fluid back into the jack, returning the flaps to their original position. This method is an advantage over the relief valve, which leaves it to the pilot to again lower the flaps by the use of the control lever. This is annoying in gusty weather, which may be continually over-loading the flaps.

An emergency extension device often used to permit the undercarriage to be lowered should the hydraulic system fail, consists of a bottle of compressed air. When the air is released it flows into the upper cylinders of the jacks, extending the undercarriage.

Another device with the same end in view, but with the added advantage of negligible increase in weight, is illustrated in Fig. 61 (*h*). It consists of a hydraulic jack, normally used for retraction, having a central chamber *A* charged with air under considerable pressure. Should the normal operating means fail, the pilot can blank off the hydraulic system and place the air reservoir in direct communication with the hydraulic chamber *E*, by means of control *C*. The compressed air will then operate the jack and lower the undercarriage.

The Tail Skid

The tail skid has very much the same functions as the undercarriage, but is subjected to very much smaller loads, and less care is taken to reduce "bounce"; only on large machines is an oleo leg fitted.



(*By courtesy of "Aeroplane"*)

FIG. 62 (*a*)

Fig. 62 (*a*) shows a non-tracking type used in the Blackburn *Bluebird*; it consists of a skid pan, two radius rods, and compression spring fitted inside the stern post. This type has an additional function to that of the undercarriage, in that the friction between the pan and the ground helps to pull up the aeroplane. Where wheel brakes are used on the undercarriage a wheel may take the place of the skid pan (the wheel being free to castor): an example is given in Fig. 62 (*b*) The skid may be steerable from the rudder controls.

Tricycle Undercarriage

It is likely that the "tricycle" type of undercarriage will be used much in the near future as an aid to high-speed landing. It consists of the usual main landing wheels set back behind the C.G., and a forward castoring wheel.

The main advantages of this type are that brakes may be applied harder without risk of overturning, stability of yaw in a cross-wind landing, and elimination of "float," as angle of attack is automatically decreased on touching down. The disadvantages are structural in that it is very difficult to provide room for retraction, especially on a single-engined machine. Also it will put up weight, as it is extremely unlikely that much weight can be saved on the tail end of the fuselage due to the elimination of the tail-skid loads.

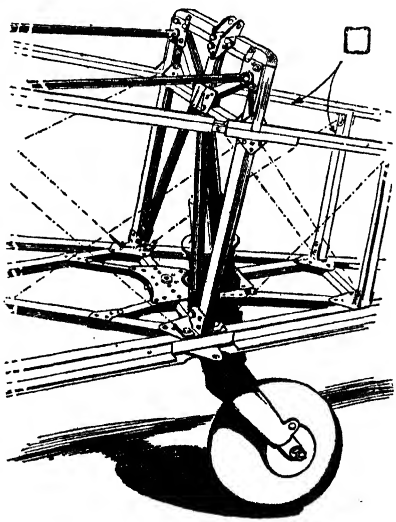
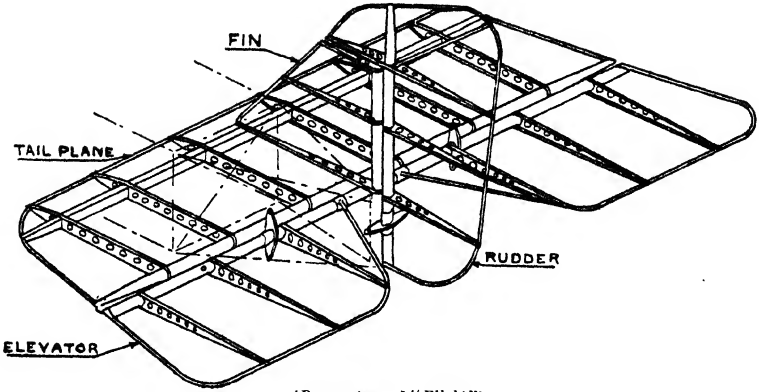


FIG. 62 (*b*)

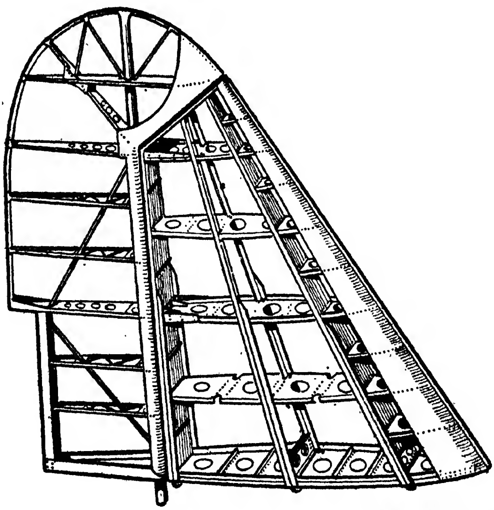
Tail Unit

The tail unit is attached to the tail end of the fuselage. It is divided into two parts; the fin and the rudder in the vertical plane, and the tail plane and elevators in the horizontal plane, giving directional and longitudinal stability and control respectively. The construction is very similar



(By courtesy of "Flight")

FIG. 63



(From "Aircraft Production")

FIG. 63 (*a*). STRUCTURAL DETAILS OF THE "BATTLE" FIN AND RUDDER

to that of the main planes and ailerons, consisting of either spars, ribs, bracing, leading and trailing edges as shown in Fig. 63, or stressed-skin construction. Fig. 63 (*a*) shows the *Battle* fin and rudder; the fin has a duralumin skin, and the rudder fabric.

The rudder and elevator should be designed with the bulk of the weight as far forward as possible, to aid mass balancing. It is for this reason that when the stabilizers are metal-covered the control surfaces often use fabric.

CHAPTER VII

FORCES IN THE STRUCTURE

MAIN PLANES

Load Distribution Along the Wing

THE intensity of loading is not constant along the wing span, but falls off towards the tip, and due to body interference. It is also different for the top and bottom planes of a biplane.

Curves of load distribution for rectangular biplane and monoplanes with shaped tips extending inboard not more than 1·2 chords are given in Fig. 64. This may be considered the same for all tip shapes. For portions cut away other than the tip, loading is reduced in the same

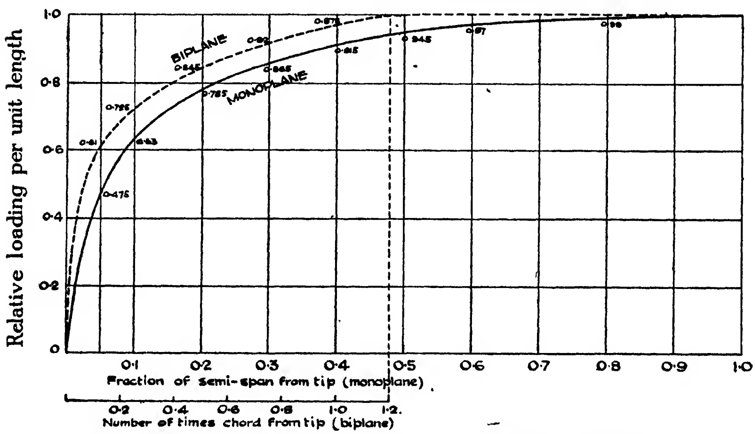


FIG. 64. LOAD DISTRIBUTION OVER RECTANGULAR WING

ratio as the chord. Tapered and twisted wings need special treatment (see Air Publication 970).

Wing Loading

The wing loading (lift per unit area) for the top and bottom planes of a biplane may be found in the following manner—

$$\text{Lift of top plane} = \tfrac{1}{2}C_{Lt}\rho S_t V^2$$

$$\text{Lift of bottom plane} = \tfrac{1}{2}C_{Lb}\rho S_b V^2$$

where C_{Lt} and C_{Lb} are the lift coefficients of the top and bottom wings respectively and S_t and S_b their respective wing areas.

$$\text{Total lift} = \tfrac{1}{2}\rho V^2 (C_{Lt}S_t + C_{Lb}S_b)$$

Let $C_{Lt}/C_{Lb} = x$.

Then $\text{Total lift} = \frac{1}{2}\rho V^2 (C_{Lb}xS_t + C_{Lb}S_b)$
$= \frac{1}{2}C_{Lb}\rho V^2 (xS_t + S_b)$

$$\text{Loading of bottom wing} = \frac{\text{Lift of bottom wing}}{S_b}$$
$$= \frac{1}{2}C_{Lb}\rho V^2$$
$$= \frac{\text{Total lift}}{(xS_t + S_b)}$$

In the same way—

$$\text{Loading of top wing} = \frac{x \text{ total lift}}{(xS_t + S_b)}$$

If the aerofoils are similar and set at the same angle, $x = 1{\cdot}2$.

Distribution of Load Along the Spar

In order to find the distribution of load along the spar, we must first subtract the weight of the wing structure from the total lift or weight of the aeroplane in horizontal flight.

Net load on wings = Weight − Weight of wing structure = W_n

From above—

Net loading on top plane $= x \,.\, W_n/(xS_t + S_b)$

Net loading on bottom plane $= W_n/(xS_t + S_b)$

For a monoplane, net loading $= W_n/S$

Let length of semi-span = s ft., and chord = C ft.

For rectangular biplanes, effective shortening of semi-span due to tip losses = $0{\cdot}2C$, and for a rectangular monoplane = $0{\cdot}125s$.

Net load on semi-span, top wing = Loading × $S_t/2$
$= xW_n \,.\, S_t/2(xS_t - S_b)$

Max. load per foot run, top wing = Net load/Effective semi-span
$= xW_n \,.\, S_t/2(xS_t + S_b)\,(s - 0{\cdot}2C)$

and for bottom wing $= W_n \,.\, S_b/2(x \,.\, S_t + S_b)\,(s - 0{\cdot}2C)$

Max. load per foot run for a monoplane $= W_n/2 \times 0{\cdot}875s = W_n/1{\cdot}75s$

To find the load per foot run at any position along the span, multiply the ordinates of the load distribution curve by the maximum load per foot as found above.

To find the load per foot run along the spars, draw a plan form of the wing to any convenient scale showing the positions of the spars (Fig. 65 (a)). On it draw the locus of the centre of pressure, which may be assumed to be a constant proportion of the chord from the leading edge, for the whole span, irrespective of the shape of the wing tip.

Divide the load curve and the wing plan into a suitable number of divisions along the span. At each division find the value of y/x and multiply it by the height of the load curve at that point, which gives the load per foot on the rear spar for the point.

By plotting the values the load curve for the rear spar is found. That for the front spar is found by subtracting the rear spar value from the whole.

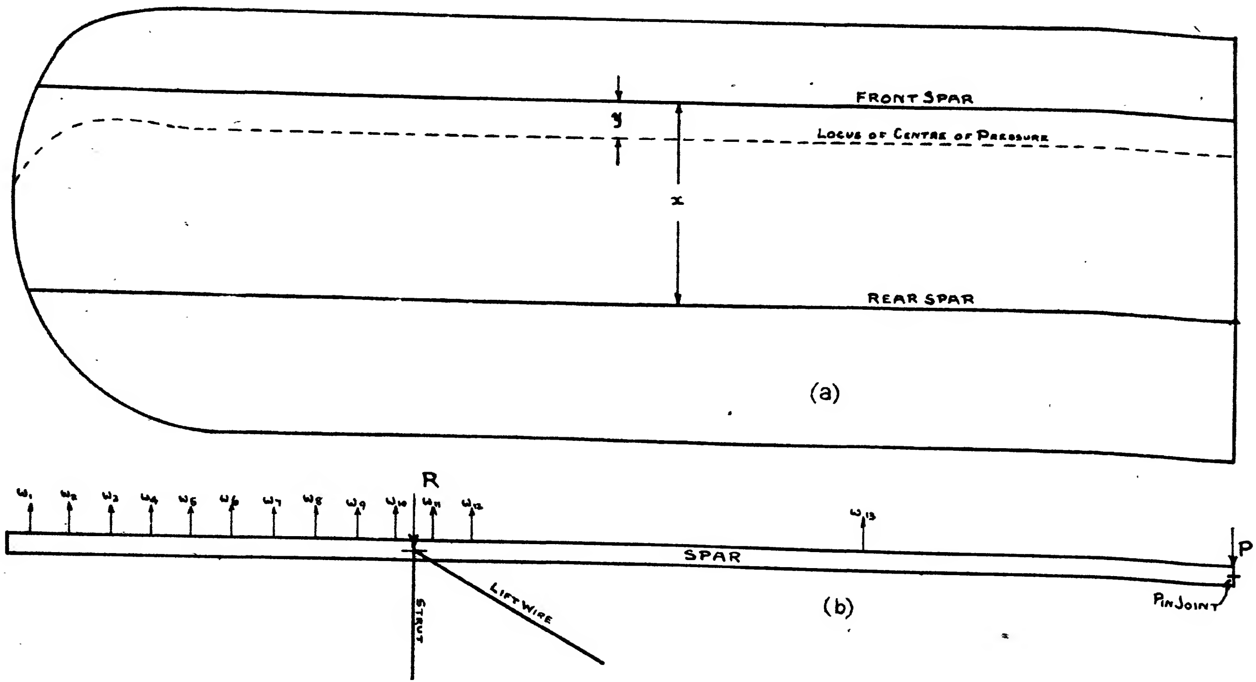

FRONT SPAR
LOCUS OF CENTRE OF PRESSURE
REAR SPAR
y
x
(a)
R
w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10} w_{11} w_{12} w_{13}
P
SPAR
STRUT
LIFT WIRE
PIN JOINT
(b)

FIG. 65

Lift Reaction

Read the actual loading from the front spar load curve at the centre of each division and multiply this by the actual length of span represented by each division. This will give the load on each division assumed concentrated at the centre.

Let w_1, w_2, w_3, etc. (Fig. 65 (*b*)), represent these loads, and l_1, l_2, l_3, etc., their distances from the pin-jointed wing attachment fitting.

If P is the reaction at the attachment fitting, R the reaction at the outer strut, and d the distance of R from P,

then $\qquad R + P = w_1 + w_2 + w_3 + \text{etc.} \quad = W$

and $\qquad w_1 l_1 + w_2 l_2 + w_3 l_3 + \text{etc.} - Rd = 0,$

from which R and P may be found.

The rear spar is treated in a similar manner.

The above is written to give a clear understanding of what is meant by the lift reaction at the joints. As explained in Chapter V, it is not intended to make any attempt to find the size of or the stresses in an aeroplane spar due to the complications arising from the effect of end loads and more than two supports. It is, however, possible to compare the effect of different types of wing bracing on the strength/weight ratio and head resistance.

Example. A single-bay biplane with rectangular wings has a semi-span of 20 ft. and a chord of 7 ft. for the whole span. Both wings are identical in shape and setting, but the effective area of the bottom wing is reduced by 25 sq. ft., due to the fuselage and its interference. The front and rear spars are 12 in. and 4 ft. 6 in. back from the leading edge respectively, and the C.P., for the case considered, is at $\frac{1}{4}$ chord.

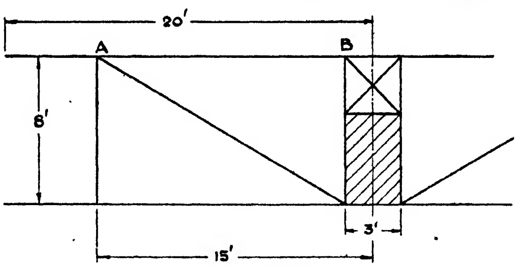


Fig. 65 (*c*)

The total weight of the aircraft is 10,600 lb. and the wings weigh 1,600 lb.

(*a*) Plot the load curve for the top front spar.

(*b*) If the front truss is as in Fig. 65 (*c*), find the compression in the front spar AB due to the lift reactions.

Top Wing

$$\text{Max. load/ft.} = \frac{xW_n \cdot S_t}{2(xS_t + S_b)\,(s - 0{\cdot}2C)}$$

$$= \frac{1{\cdot}2\,(10{,}600 - 1600)\,40 \times 7}{2(1{\cdot}2 \times 40 \times 7 + 40 \times 7 - 25)\,(20 - 0{\cdot}2 \times 7)}$$

$$= 137{\cdot}5$$

$$\therefore \begin{array}{l}\text{Max. load/ft.}\\ \text{on front spar}\end{array} = \frac{137{\cdot}5 \times 33}{42} = \underline{\underline{108 \text{ lb./ft.}}}$$

Bottom Wing

$$\text{Max. load/ft. on front spar} = \frac{W_n \cdot S_b}{2(xS_t + S_b)(s - 0{\cdot}2C)} \times \frac{33}{42}$$

$$= \frac{9000 \times 255}{1180\ (18{\cdot}5 - \tfrac{3}{4} - 0{\cdot}2 \times 7)} \times \frac{33}{42}$$

$$= \underline{\underline{91}}\ \text{lb./ft.}$$

To get load curve of top spar, multiply ordinates of biplane load curve (Fig. 60) by 108. This is given in Fig. 65 (d). The area under this curve

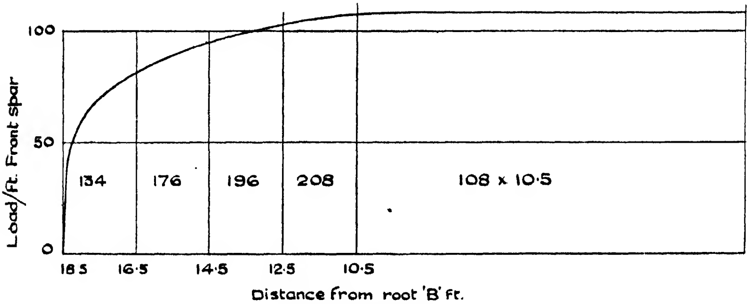


FIG. 65 (d)

represents the total load on the spar. Split the area into a suitable number of parts, and multiply the areas by the distance of their centroids from B, sum the results, and divide by distance AB, which gives the reaction at A—

Reaction

$$= \frac{134 \times 17{\cdot}5 + 176 \times 15{\cdot}5 + 196 \times 13{\cdot}5 + 208 \times 11{\cdot}5 + 108 \times 10{\cdot}5 \times 5{\cdot}25}{13{\cdot}5}$$

$$= 1{,}198\ \text{lb.}$$

Reaction at A bottom spar $= 1198 \times \frac{91}{108} = 1010$ lb.

Total upward reaction at A $= 1198 + 1010$

$= 2208$ lb.

Compression force in AB $= \dfrac{2208 \times 13{\cdot}5}{8}$

$= \underline{\underline{3720}}$ lb.

Aspect Ratio

In finding the effect of aspect ratio it will be best to consider the complete wing structure. It is a beam carrying a distributed load. The maximum bending moment will be $\frac{Wl}{2}$, where W equals total load on half span and l equals half span.

Any increase in span will mean a proportional increase in bending moment and length of spar, and therefore an increase in weight of wing structure, provided the total wing area remains constant. Extra weight may also be required to give sufficient stiffness to prevent flutter. Increased depth may save some of the weight increase, but this leads to increased drag. For aerodynamic reasons the efficiency of the aerofoil increases with aspect ratio, a large aspect ratio means more lift and a smaller induced drag.

There are therefore two effects of increasing the aspect ratio—

(*a*) To increase the weight due to increased bending moment and length, and to give stiffness.

(*b*) Increased aerodynamic efficiency.

The relative importance of these will vary. If the aspect ratio is large, (*a*) will have the greater effect; and if small, the effect of (*b*) will be greater. It follows that the most efficient aspect ratio must be neither large nor small, and normally varies between 5 and 6½ for monoplanes and 5½ and 9 for biplanes. For a glider, where the structure weight is relatively unimportant, it may be as high as 20. The Vickers geodetic wings use higher aspect ratios, as they resist torsion better than other types.

BRACED WING STRUCTURES

Monoplane Trusses

Fig. 66 (*a*) and (*b*) show two monoplane trusses and their stress diagrams. (*a*) will have a large compressive end load on a long spar. By bringing in the support half-way the reaction AB will be doubled, and it will be seen from the stress diagram that the end load is the same in each case. In (*b*) the stress in the inner portion of the spar will be decreased due to the shorter length, but the maximum bending moment and therefore the stress on the overhanging portion will be four times as great.

By having a high wing (Fig. 66 (*c*)) it will be seen that the end load is considerably decreased, the strut AC being fixed at a less acute angle.

If the strut is above the wing (Fig. 66 (*d*)), the end load will be tension, giving a much lighter spar. The strut, however, will be heavy, it having to take the flying loads in compression, whereas in the other cases the lift was taken in tension.

Fig. 66 (*e*) shows a pure cantilever wing, a style which is now most popular.

There are no end loads due to lift bracing in this case, but the bending moment will be very great at the root, necessitating an increased depth of spar and wing. There will be large loads due to the couple at the attachment to the fuselage.

These loads may be reduced, and the necessity of putting a member inside the fuselage to take them obviated by putting a short strut (shown dotted) to the top or bottom of the fuselage, depending on where the spar root is attached. This will also reduce the bending moment and so enable a decreased depth of spar to be used. Fig. 66 (*f*) shows the approximate bending moment diagram for a pure cantilever spar compared with that of a spar supported with a strut at A.

Biplane Trusses

Fig. 67 (*a*), (*b*), and (*c*) show three biplane trusses. It will be seen from the diagrams that (*a*) has a compressive end load in the top spar

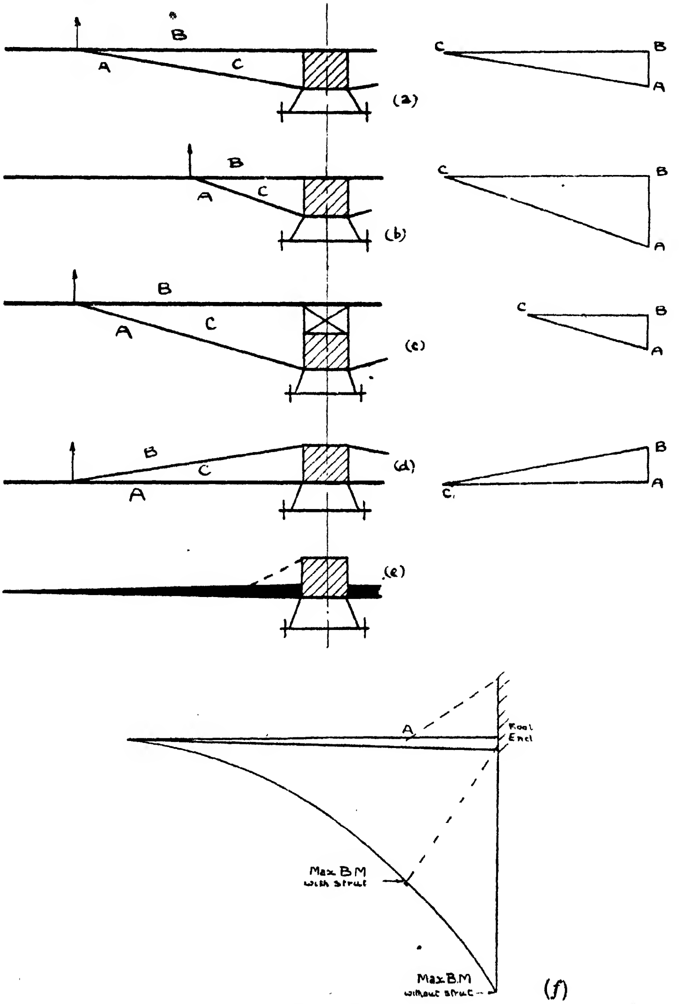


FIG. 66

and interplane strut, and no end load in the bottom spar, while (*b*) has no end load in the top spar, tension in the interplane strut and bottom spar, and a heavy compression in the diagonal strut. This means that

the spars and interplane strut of (*b*) will be lighter, but this save in weight will probably be more than counterbalanced by the need for a very heavy diagonal strut.

Fig. 67 (*c*) shows another wireless truss, where the diagonal strut taking compression has been considerably shortened and the compression reduced. There will be compression in the top spar, but as the unsupported lengths have been reduced this spar will be light compared with (*a*). An advantage of wireless external bracing is a reduction of head resistance, wires having a high resistance due to vibration.

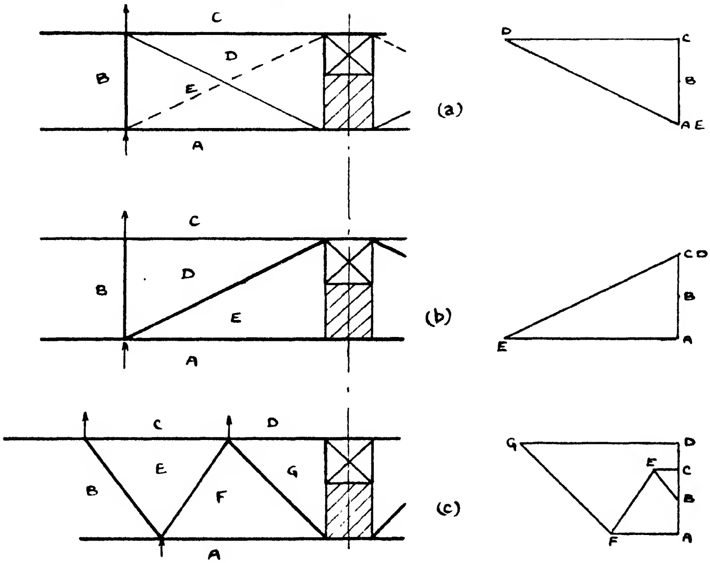


FIG. 67

Examples of two bay biplanes are shown in Fig. 68. It will be seen in each case that the loads in the inner bay are greatest.

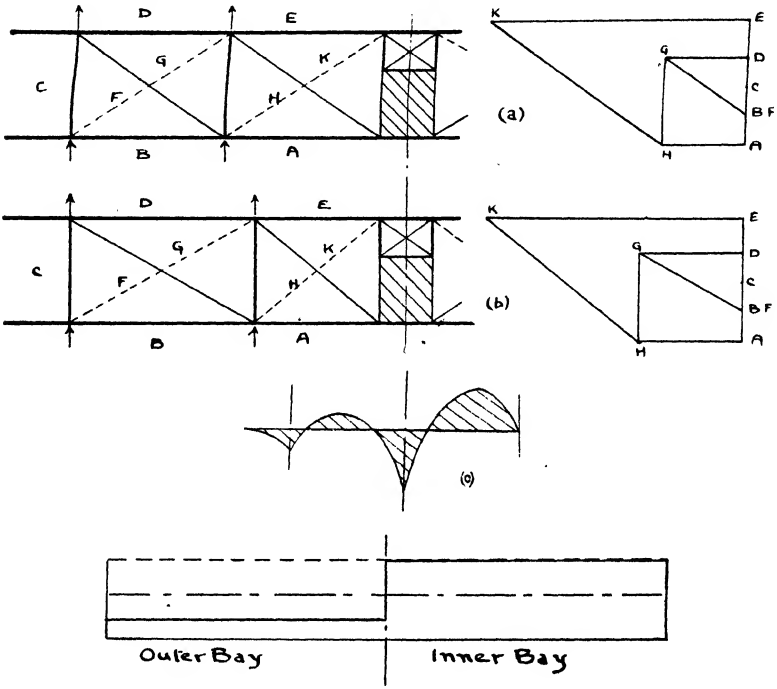
Case (*a*) shows a biplane with each bay at the same length, and as the compressive end loads are much greater in the inner bay the spar will have here to be heavy to withstand them, but the outer bay spar is only required to be comparatively light.

Biplane spars are normally made of constant section throughout, as the extra weight of fittings required to change the section to suit the bending moment, which changes something like that shown in Fig. 68 (*c*), would be as great as the weight saved on the spar itself. Thus the spar in case (*a*), being of constant section, will be much heavier than is required in the outer bay in order that it may be of the required strength in the inner bay.

In case (*b*) the reduction of length of the inner bay reduces the bending moment there and the spar may be made lighter, whilst the extra length of the outer bay will increase the strength required there. By using

suitable lengths of bays it can be so arranged that the spar requires the same strength in both bays.

This is illustrated diagrammatically in Fig. 68 (*d*). The height of the full line shows the strength (or weight) of spar required for case (*a*), and the dotted line that used for constant section, whilst the chain-



(*d*) FIG. 68

dotted line shows how the smaller inner bay (case (*b*)) will reduce the weight of spar required.

The above also applies to the longerons in the fuselage, and, where practicable, the inner bays are made shorter here.

Drag Bracing

The spars are braced together in the plane of the wing by means of drag bracing. This usually takes the form shown in Fig. 69. It will be seen from the diagram that the loads are greatest in the inner bays. These end loads will increase or decrease the spar loads due to lift, depending on whether they are of similar or different sense.

Effect of Stagger

The effect of stagger is to increase the loads in the spars, external and drag bracing, and so to increase the weight. Fig. 70 shows sections,

with and without stagger, of the wing structure in the plane of the incidence bracing. In each case the assumed lift is 200 lb. and drag 10 lb. on the front spar at this point.

It will be seen, by resolution, that the lift bracing takes 200 lb. and the drag bracing 10 lb. if there is no stagger, but 225 lb. and 100 lb. respectively if there is stagger, and the lift bracing is in the plane of the top and bottom spars.

Stagger is only adopted for practical reasons, as the gain in lift is neutralized by the increase in weight.

Students often ask why on certain aeroplanes the anti-drag wires are

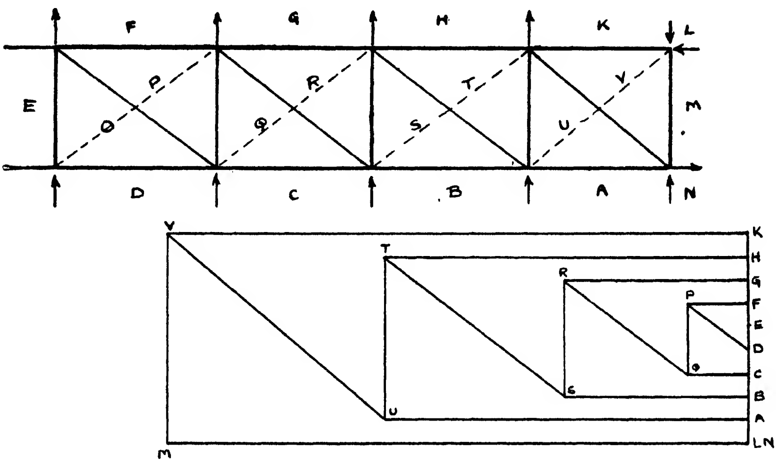


FIG. 69

of larger cross-section than the drag wires. The reason is that the drag bracing does not take drag, nor the lift bracing lift. We have seen, in the previous example, that with stagger both the lift and drag bracings take a greater force than the lift and drag respectively. Now if the stagger had been negative, and of the same magnitude, the horizontal component of the 225 lb. in the lift bracing would have been forward. This component is 90 lb. and the drag of 10 lb. would give a resultant forward force of 80 lb., thus loading the anti-drag wires in flight.

Again consider the case illustrated in Fig. 70 (*a*). The strut is staggered, but the lift wire is vertical. For simplicity the drag bracing is again assumed horizontal. At the bottom joint the 200 lb. lift puts a force of 225 lb. in the interplane strut which, owing to its augularity, has a horizontal component at 90 lb. backwards. This, together with the 10 lb. drag, puts a "drag" force of 100 lb. in the drag bracing.

At the upper joint the interplane strut load has its vertical component of 200 lb. taken by the lift wire, together with the 200 lb. lift at this joint. Its horizontal component of 90 lb. forward, reduced by the 10 lb. drag, puts an "anti-drag" force of 80 lb. in the drag bracing, so loading the anti-drag wires.

On this aeroplane there would be larger anti-drag wires in the top plane, and larger drag wires in the bottom plane.

Duplication of Flying Wires

In order to guard against complete failure of a machine, due to the breakage of a wire, all wires taking load in flight should be duplicated, either directly or indirectly.

The duplication is arranged so that if a wire is broken the structure is strong enough for slight manoeuvres but not for large accelerations.

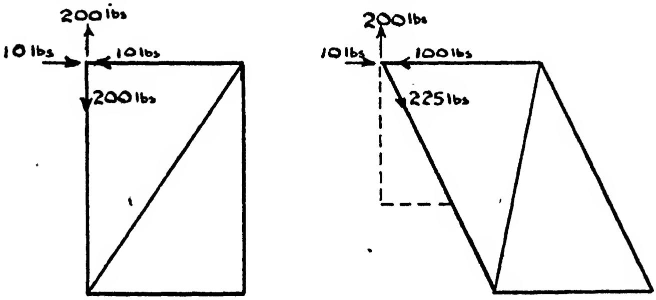


FIG. 70

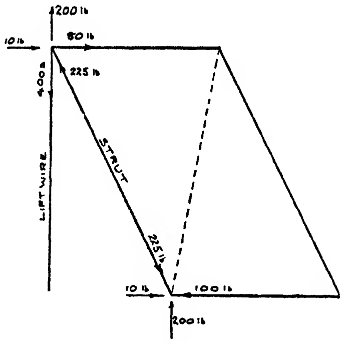


FIG. 70 (*a*)

Direct Duplication. In this method two flying wires are fitted instead of one. Each must have a separate anchorage, and be able to take two-thirds of the load that one would have taken, as it is impossible to make each take the same load. Care must be taken that one wire is not rigged tighter than the other, or it will take a greater load.

It should be noted that duplication does not increase the strength of the structure, for even if the duplicated wires could be relied upon to take $1\frac{1}{3}$ the load of a single wire, the other members, which also help to make the structure, are no stronger.

Actually if the load is more than moderately high when a wire is broken, the remaining wire is likely to break too, due to it having to take its own load plus the suddenly applied load from the broken wire.

Thus duplication only guards against failure due to breakage of a wire, due to fault, fatigue, or gunshot.

EXAMPLE. Duplicate lift wires are rigged such that one is 10 ft. 5·15 in. and the other 10 ft. 4·85 in. A total load of 6000 lb. has to be transmitted. Find the load taken by each wire, given $E = 30 \times 10^6$ lb./sq. in.

Area of each wire = 0·034 sq. in.

Elongation of short wire = elongation of long wire + 0·3 in.

$$\frac{PL}{AE} = \frac{P_1L_1}{AE} + 0{\cdot}3$$

where P and P_1 equal load in short and long wire respectively, and L and L_1 equal length of short and long wire respectively.

$$P + P_1 = 6000 \text{ lb.}$$

$$P_1 = 6000 - P.$$

Then
$$\frac{PL}{AE} = \frac{6000L_1 - PL_1}{AE} + 0{\cdot}3,$$

$$PL + PL_1 = 6000L_1 + 0{\cdot}3AE$$

$$P = \frac{6000L_1 + 0{\cdot}3AE}{L + L_1}$$

$$= \frac{6000 \times 125{\cdot}15 + 0{\cdot}3 \times 0{\cdot}034 \times 30000000}{250}$$

$$= 4228 \text{ lb.}$$

$$P_1 = \underline{1772} \text{ lb.}$$

The short wire carries a load of over two-thirds of the total, and is liable to fracture.

Indirect Duplication. The flying wires and the fuselage side bracing wires may be duplicated indirectly by means of incidence bracing and

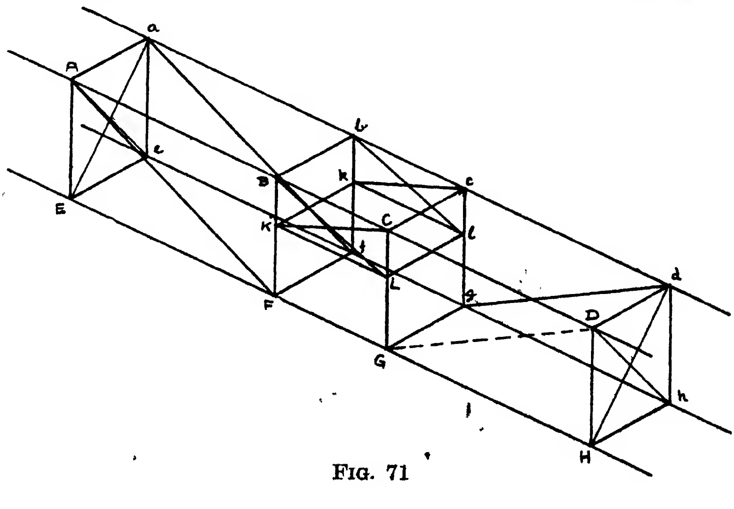


FIG. 71

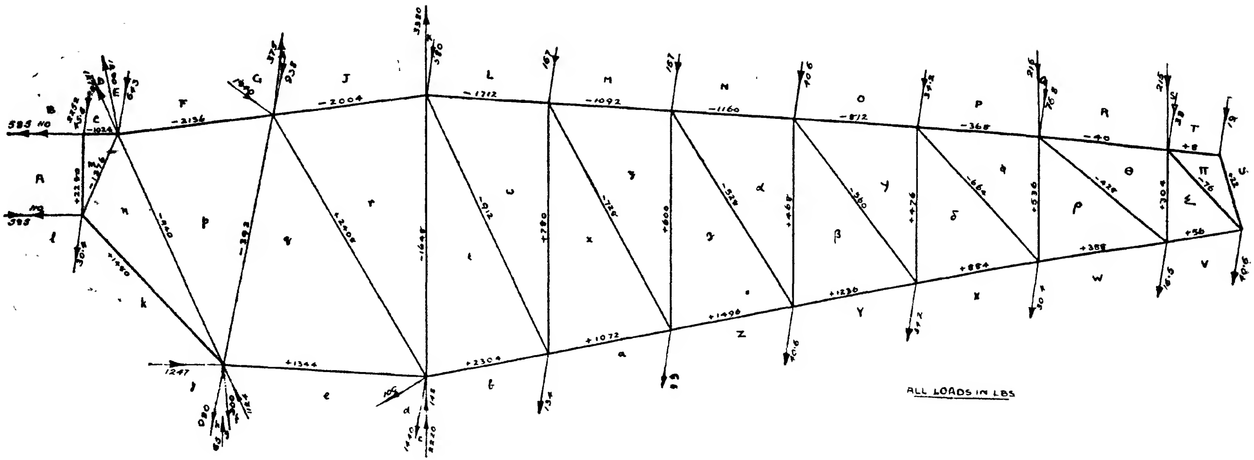


FIG. 72

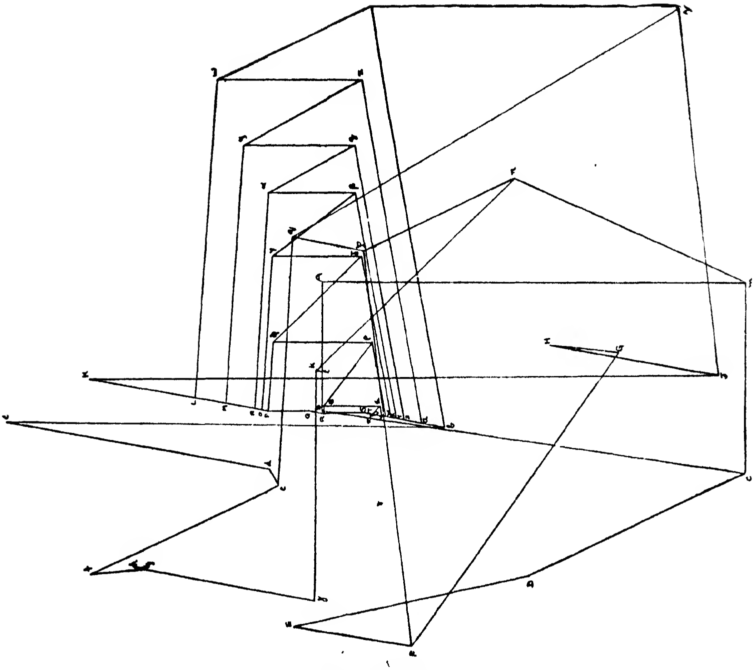
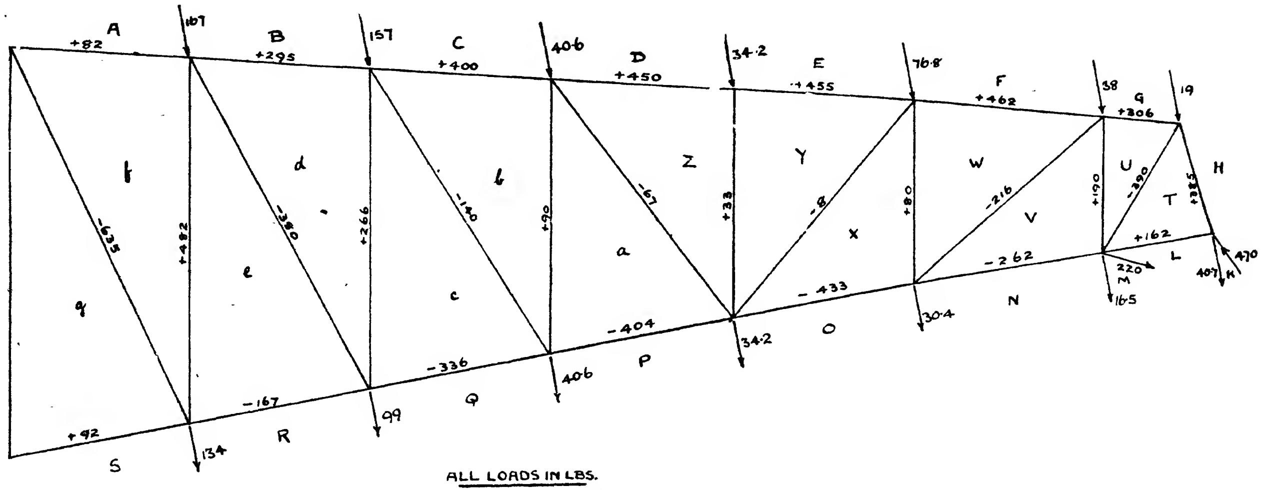


FIG. 72 (a)


A
B
C
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a
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f
g
+82
+295
+400
+450
+455
+462
+306
+385
+162
-262
-433
-404
-336
-167
+92
-635
+482
-380
+266
-140
+90
-67
+33
-8
+80
-216
+190
-390
167
157
40·6
34·2
76·8
38
19
470
407
220
16·5
30·4
34·2
40·6
99
134
ALL LOADS IN LBS.

FIG. 73

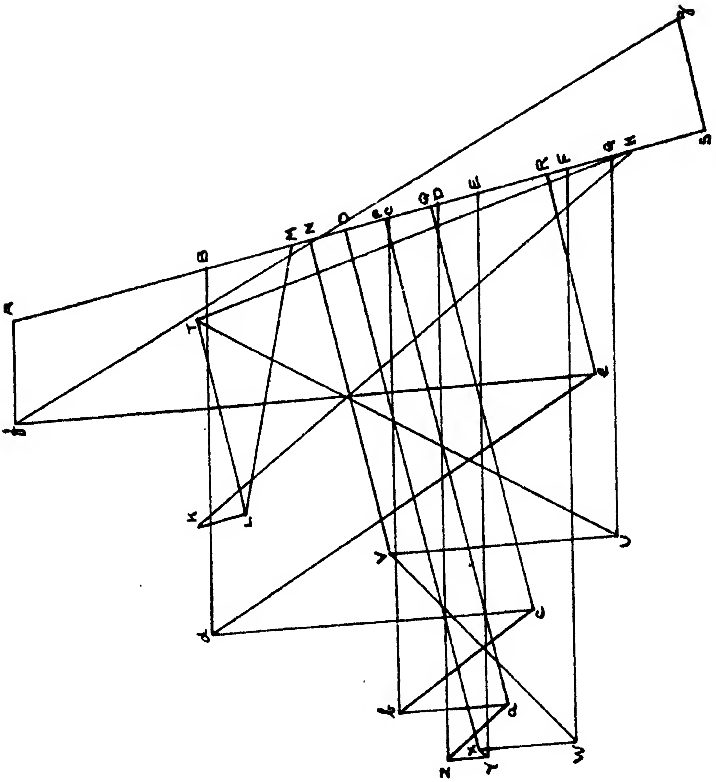


FIG. 73 (a)

the centre cross-bracing respectively. Referring to Fig. 71, suppose the flying wire *DG* breaks. The total reaction at *D* has then to be taken by the incidence wire *Dh*; this will transfer the reactions from the front to the rear truss, which must now take a greater load.

The result of this will be—

1. To put an extra load in the top and bottom drag bracing, due to the load in *Dh*.

2. Increase the tension in *dg* and compression in *dh*.

3. *To alter the end loads in the spars, so that the port loads are no longer balanced by equal and opposite starboard loads.*

The effect of (3) will be—

(*a*) To cause the spar *AD* to tend to move in the direction *AD*. Extra loads in drag bracing and *CK*.

(*b*) To cause the spar *ad* to tend to move in the direction *da*. Extra loads in drag bracing, *bl* and *af*.

(*c*) To cause the spar *FH* to tend to move in the direction *HF*. Extra loads in drag and centre section bracing.

(*d*) To cause the spar *eg* to tend to move in the direction *eg*. Extra loads in drag, centre section bracing, and *af*.

The same effects will take place if, due to faulty rigging, the flying wire *DG* is slack and the corresponding incident wire *Dh* and rear flying wire *dg* are tight. If one incidence wire is tighter than the other, the rigger may be sure that one truss is taking more than its share of the load.

THE FUSELAGE

The fuselage acts as a beam supported at the wings in flight. In landing it is supported by the undercarriage and tail-skid, and must withstand the weight of the whole structure in a fairly bad landing.

In flight it must be strong enough to withstand the following loads—

Up load on tail (C.P. forward).
Down load on tail (C.P. back and nose dive).
Side load on fin and rudder (in a turn).
Torsion (in a turn, when the fin and rudder are unsymmetrically placed about the centre).
Weight of structure and load not in the wings.
Engine thrust and torque (unless the engines are in the wings).

When finding the internal loads the fuselage may be treated as a whole or regarded as consisting of three portions, viz. front, rear, and centre portions.

The front and rear portions are cantilevers transferring the forward and tail loads respectively to the centre portion, which is supported by the wings or undercarriage.

Fig. 72 shows the stress diagram and loads in a fuselage treated as a whole for the C.P. back case, and Fig. 73 the rear portion of the same fuselage for the landing case.

The loads on one side only are taken, the other side being identical.

In making these diagrams the structure is assumed pin-jointed, and the loads carried and weight must be distributed over the joints and will act vertically downwards. This does not mean they act perpendicularly to the datum line of the fuselage, but at an angle depending on the attitude of the machine for the cases taken.

The fuselage should be stressed for the following conditions of loading—

Front Portion.	Landing with engine off.
	Normal flight C.P. forward.
	Turning in flight.
	Thrust and torque.
Rear Portion.	Landing on wheels and skid.
	Normal flight C.P. forward.
	Nose diving.
	Side load on fin and rudder.
Centre Portion.	Loads from remainder of structure for all conditions.

If, as is usual, the fin and rudder are not symmetrically placed about the centre line of the fuselage, the side load in a turn will put the fuselage in torsion. The method of finding loads in this case is far too involved to be included in this book. Note that the torsion is taken by the side bracing, and not the internal cross-bracing.

So far we have only considered the girder type; the monocoque fuselage may be treated as a tubular beam. As, however, such calculations as can be made are not very reliable, due to elastic instability causing failure, an actual fuselage should be tested by loading it to the maximum load it will have to stand. For general considerations, see Chapter VI.

THE UNDERCARRIAGE

Springing

In normal aircraft the vertical travel of the undercarriage from a ground reaction equal to the static weight of the aircraft, to a ground reaction corresponding to three times the weight, should not be less than 4 in. plus the deflection of the tyres. It should also be such that the total work of compression is not less than the kinetic energy at the moment of impact, i.e.

$$\frac{Wv^2}{2g} \text{ ft.-lb.}$$

where W = weight of aircraft in lb.
and v = vertical velocity of descent in ft./sec.
v for civil aircraft may be taken as $(3 + 0{\cdot}1V)$, where V is stalling speed in ft./sec.

The work done must be found from a load-deflection graph of the shock-absorber, tyres, and axle. An example for an undercarriage using compression rubbers is given in Fig. 74. For a contraction of 8 in. and load of 6000 lb. the work done is given by the shaded area multiplied by the scale used.

Example 1. A civil aeroplane weighing 1500 lb. has a stalling speed of 50 ft./sec. If the load-deflection graph for the undercarriage is as Fig. 74, find the maximum ground reaction and the full deflection of the undercarriage.

$$\text{Work done} = \frac{Wv^2}{2g}$$

$$v = (3 + 0{\cdot}1 \times 50) = 8 \text{ ft./sec.}$$

$$\text{Work done} = \frac{1500 \times 64}{64}$$

$$= \underline{1500 \text{ ft.-lb.}}$$

The area of the graph must therefore equal 1500 ft.-lb.
This may be found by trial and error.
Shaded area represents 1540 ft.-lb.
This is 40 ft.-lb. too large.

Divide the graph divisions into tenths (not shown in Fig. for clearness).

Then as each small square represents $3\frac{1}{3}$ ft.-lb., the area must be $\frac{40}{3\frac{1}{3}} = 12$ small squares smaller. Deduct these and the correct area ABC is found.

Then $BC = 5800$ lb. is the ground reaction,

and $AC = 7{\cdot}9$ in. is the full deflection.

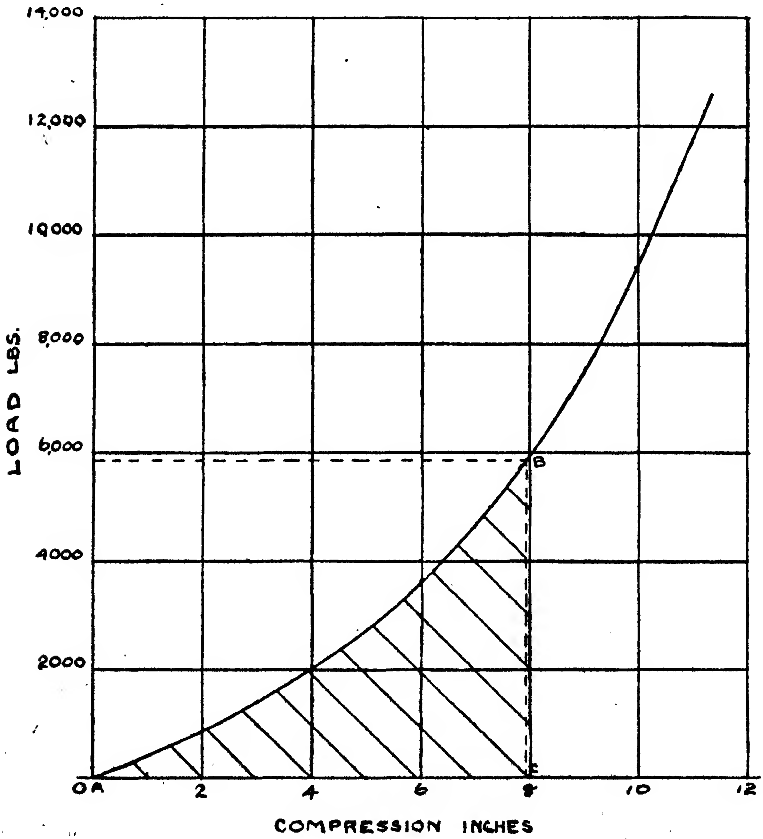


FIG. 74

Strength

The strength of the undercarriage, and the rest of the structure, should be investigated for the following cases—

1. The aircraft landing on the wheels with the thrust line horizontal.
2. The aircraft landing so that the wheels and skid touch the ground simultaneously.
3. A side load applied to the hub of each wheel equal to the weight of the aircraft divided by the number of landing wheels.
4. Aircraft stationary on the ground with the wheels chocked, skid at ground level, and engine developing maximum horse-power.
5. Aircraft landing as in Case 1, with an horizontal drag force applied at the hub equal to the ground reaction divided by 4.

The ground reactions for Cases, 1, 2, and 5 are determined from the springing characteristics of the undercarriage as shown in the previous example.

In Case 1 the total ground reaction is divided evenly between the wheels, the skid taking no load. In addition there will be a vertical load on the wings, distributed as for the C.P. forward case, and an upload on the tail, at $\frac{1}{3}$ tail plane chord from the leading edge, to give balance. The total upload for the wings and tail plane is equal to the weight of the aircraft. The load factor to be 1·15 for alighting gear and 1·25 for the rest of the structure.

For Case 2 the total ground reaction may be as Case 1, and the same load factors used, or the machine may be considered standing on the ground with the weight divided between the wheels and skid, and a load factor of 4 used for alighting gear and 4·5 for the rest of the structure. In either case the reactions on the wheels and skid may be found from the total reaction by a simple moment equation if the position of the C.G. is known.

In Case 3 the side load is considered to be the only external force acting, and is balanced by the inertia forces. The load factor is 0·7.

In Case 4, thrust may be taken as 5 lb. per h.p. No air forces are considered. Load factor equals 2.

Case 5 is as Case 1, with the addition of the hub load.

Note. The load factors given are for civil aircraft. For service aircraft they are slightly higher. In the event of a bad landing, it is arranged, by using smaller load factors for the undercarriage, that it will fail before the rest of the structure.

The size of the wheels should be such that

$$\frac{W}{n} = 13Db$$

where W = weight of aircraft, lb.
n = number of landing wheels
D = overall diameter of wheels, inches
b = tyre width, inches.

The loads in the members of the undercarriage for the above cases can be found by simple statics for the single-axle undercarriage, and although this is now little used, it not being retractable, it provides a useful example in statics. The method for divided undercarriages is given by Mr. A. E. Russell in *Flight*, 28th November, 1930, but is too

advanced for inclusion here. The loads in the rest of the structure are found by stress diagrams.

Example 2. Find the design loads in the members of the undercarriage of the aeroplane for which the ground reaction was found in Example 1.

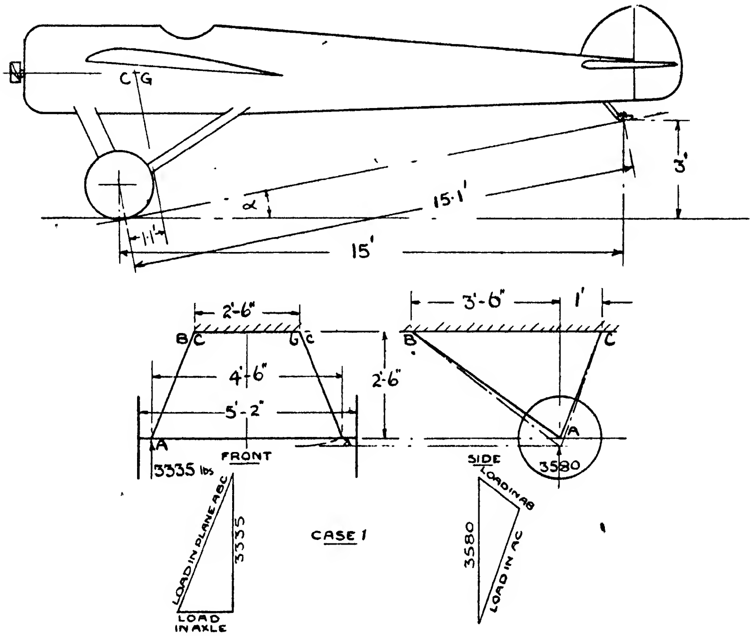


Fig. 75 (*a*)

Given max. h.p. equals 140. Line drawings of the aeroplane and undercarriage, with the thrust line horizontal, are given in Fig. 75 (*a*).

Case 1.

$$\text{Total ground reaction as found in Example 1} = 5800 \text{ lb.}$$

$$\text{Ground reaction per wheel} \times \text{load factor} = \frac{5800 \times 1{\cdot}15}{2}$$

$$= \underline{3335 \text{ lb.}}$$

This force is also at joint A, since the axle is continuous, and it puts a maximum bending moment in the axle at A, equal to

$$\frac{(5 \text{ ft.} - 2 \text{ in.}) - (4 \text{ ft.} - 6 \text{ in.})}{2} \times 3335$$

$$= 4 \times 3335$$

$$= \underline{13{,}340 \text{ in.-lb.}}$$

To find the loads in the members draw the triangle of forces for the joint A in the front view.

This gives—

3580 lb. in the plane ABC

and 1320 lb. tension in the axle.

Next obtain the true view of ABC (shown dotted), and draw the

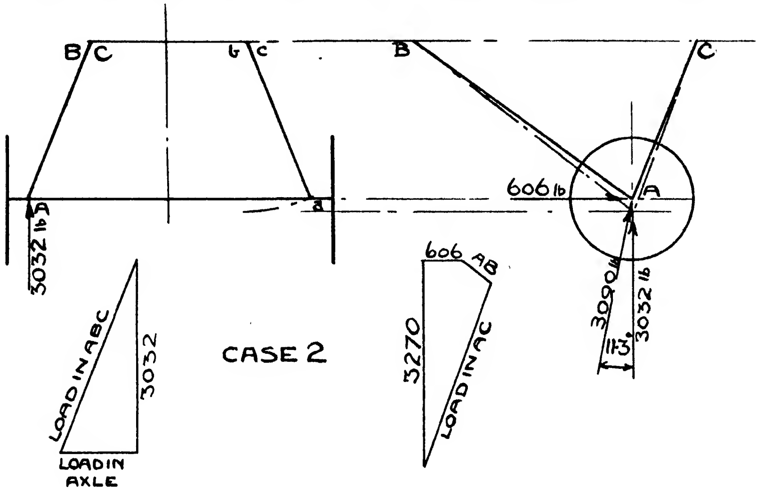


FIG. 75 (b)

triangle of forces for the joint A in the plane ABC. The external force at A being the 3580 lb.

This gives—

Load in AB = 1320 lb. compression.

„ „ AC = 2960 lb. „

Case 2.

Total ground reaction = 5800 lb. (as before).

Find reaction at wheels by taking moments about skid.

$$5800\,(15{\cdot}1 - 1{\cdot}1) - 15{\cdot}1R = 0$$

$$R = \frac{5800 \times 14}{15{\cdot}1}$$

$$= 5377 \text{ lb.} = \text{reaction at wheels.}$$

Reaction at each wheel × load factor

$$= \frac{5377 \times 1{\cdot}15}{2} = \underline{3092} \text{ lb.}$$

This reaction acts at an angle a to that in Case 1.

$$\text{Tan } a = 3/15 = 0{\cdot}2.$$

$$a = \underline{11{\cdot}3^\circ}$$

The reaction of 3092 lb. acting at 11·3° may be resolved into an horizontal component and vertical component at A (see side view, Fig. 75 (b)).

$$\text{Horizontal component} = 3092 \sin 11{\cdot}3 = 606 \text{ lb.}$$

$$\text{Vertical component} = 3092 \cos a = 3032 \text{ lb.}$$

This vertical component will put a load in the axle, also it is not in the plane ABC. Draw the triangle of forces for the joint A in the front view, using this load.

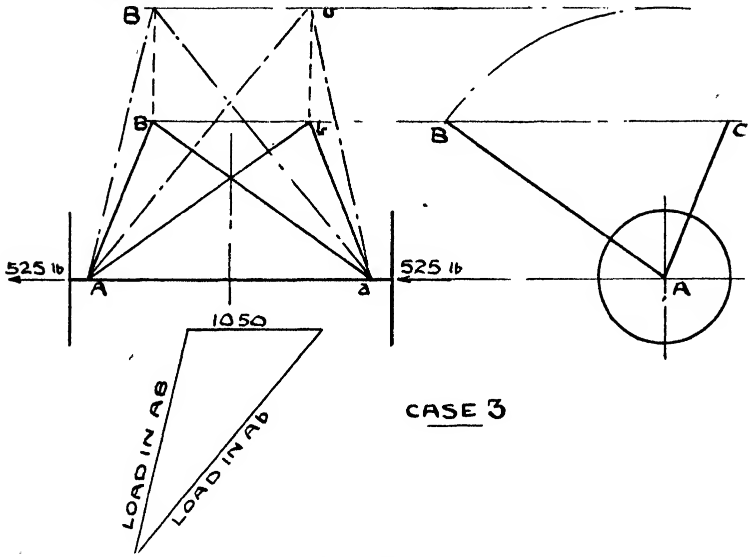


Fig. 75 (c)

This gives—

3270 lb. in plane ABC

and 1200 lb. tension in the axle.

Next draw the polygon of forces for the joint A in the plane ABC, the external loads being 606 lb. horizontal and 3270 lb. vertical.

This gives—

Load in AB 590 lb. compression.

" " AC 3100 lb. "

Case 3.

Weight of aircraft = 1500 lb.

Side load on each wheel × load factor

$$= \frac{1500 \times 0{\cdot}7}{2} = 525 \text{ lb.}$$

Consider the load is to the left.

The load of 525 lb. at a will put 525 lb. compression in the axle, which will transmit it to A, where the two 525 lb. loads are taken by the radius rod AB, and bracing wire Ab. AB being in the plane of the radius, rods AC will take no load. This case is the criterion for the strength of the bracing wires.

Draw the true view of the plane $ABba$ (shown dotted, Fig. 75 (c)), then draw the triangle of forces in this plane for joint A, using the total side load 1050 lb.

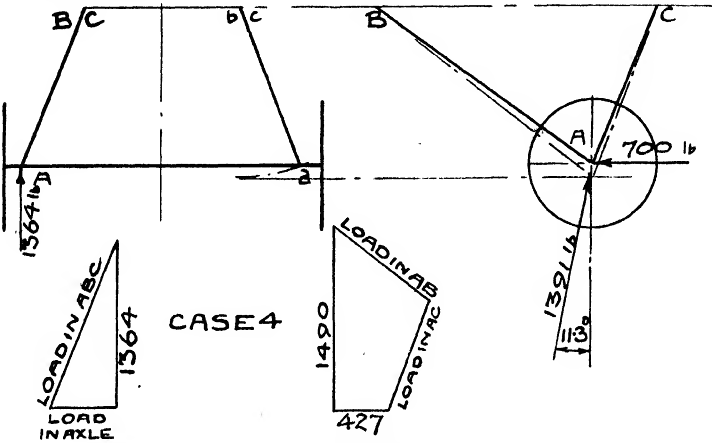


FIG. 75 (d)

This gives—

Load in AB = 1880 lb. compression.
„ „ Ab = 2360 lb. tension.

Case 4.

Thrust at 5 lb./h.p. $= 140 \times 5$
$= 700$ lb.

Vertical reaction on wheels with skid at ground level

$$= \frac{1500 \times 14}{15 \cdot 1} = 1391 \text{ lb.}$$

The wheels rotate slightly until the resultant reaction from the ground and chocks is passing through the centre line of the axle. This resultant is equal to a force equal and in the opposite direction to the thrust, and a force equal to the vertical ground reaction. This latter is at 11·3° to the vertical plane in Fig. 75 (d).

Force at each hub × load factor

= 700 lb. in horizontal plane
and 1391 at 11·3° to vertical plane.

The latter may be split up into two components, viz.—

Vertical component = 1391 cos 11·3°
= 1364 lb.
Horizontal component = 1391 sin 11·3°
= 273 lb.

This gives a total horizontal force at A equal to

$$700 - 273 = 427 \text{ lb.}$$

and a vertical force of 1364 lb.

Treating this in the same way as Case 2, we get—

A load in the axle	=	550 lb. tension.
„ „ ABC	=	1490 lb. „
„ „ AB	=	960 lb. compression.
„ „ AC	=	940 lb. „

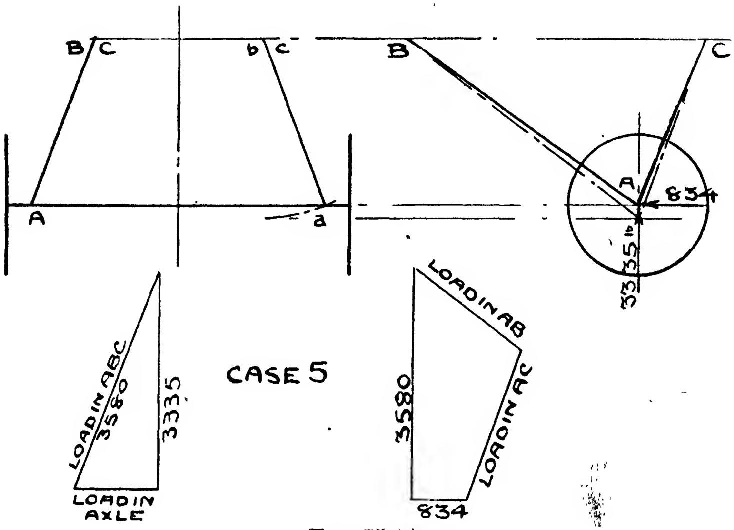


FIG. 75 (e)

Case 5.

$$\text{Drag load at hub} = \frac{3335}{4} = 834 \text{ lb.}$$

$$\text{Vertical load at hub} = 3335 \text{ lb.}$$

This is treated in the same way as Case 2—

Load in the axle	=	1320 lb. tension.
„ „ AB	=	2130 lb. compression.
„ „ AC	=	2420 lb. „

From the above we get the required strength of members from the maximum loads considering all cases—

Axle Aa	=	1320 lb. tension and 525 lb. compression.
Leg AC	=	3100 lb. compression.
Radius rod AB	=	2130 lb. compression.
Bracing wire Ab	=	2360 lb. tension.

The axle also has to take bending. Case 5 gives the maximum bending moment. The two loads of 834 lb. and 3335 lb. may be found by resolution to equal a load of 3408 lb., giving a maximum bending moment at A

$$= 3408 \times 4$$
$$= \underline{13632} \text{ in. lb.}$$

As the leg AC varies in length the calculations should also be carried out with the leg compressed under the full factored load. The amount it is compressed under this load may be found from a load-deflection graph of the leg.

CHAPTER VIII

DETAIL DESIGN

THE SPARS

In order to get economy from the weight standpoint, the spars should be made as deep as practicable. Their depth will be dependent upon the aerofoil used and their position in the aerofoil.

Due to the large proportion of loading on the front of the wing, the front spar should be as near the leading edge as possible, without appreciable loss in depth, i.e. just in front of the point of maximum aerofoil depth. The rear spar should be spaced as widely as practicable from the front spar, in order that the loads due to the couple in a nose dive may be a minimum and to give rigidity to the rear of the wing. This will mean an unavoidable reduction in spar depth, due to the taper of the rear portion of the aerofoil. The actual position will be a compromise between reduction of load and a reduction of depth.

Types

Fig. 76 shows an Armstrong spar as used on the Atlas machine. It is made from steel strip varying from 0·01 in. to 0·02 in. thick, and is $5\frac{1}{2}$ in. deep. Although weighing only 0·855 lb. per foot, it has a

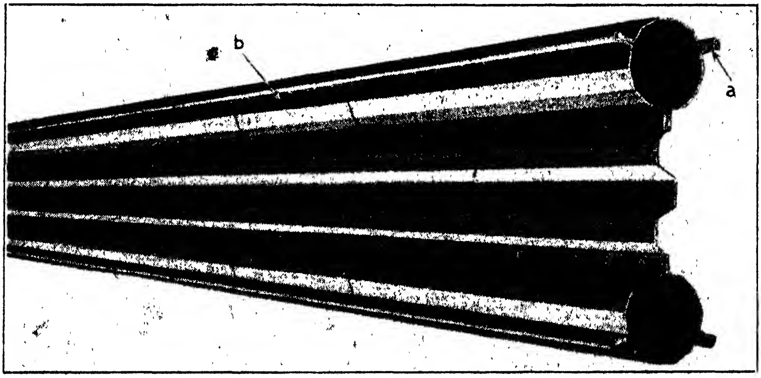


FIG. 76. ARMSTRONG "ATLAS" SPAR

moment of resistance of 28 tons-inches, and will withstand a stress of 80 tons/sq. inch in the flanges. It will be noted that the free edges (*a*) at points of high stress are covered by a small section in order to prevent failure from elastic instability.

The width of the flats in the webs are thirty-two times the thickness, but as this part has only to withstand a compressive stress of 50 tons/sq. inch wave formations will not occur.

Local reinforcements may be made by tubular inserts and extra flanges as shown at (*b*).

The method of fixing the ribs to these spars is illustrated in Fig. 77.

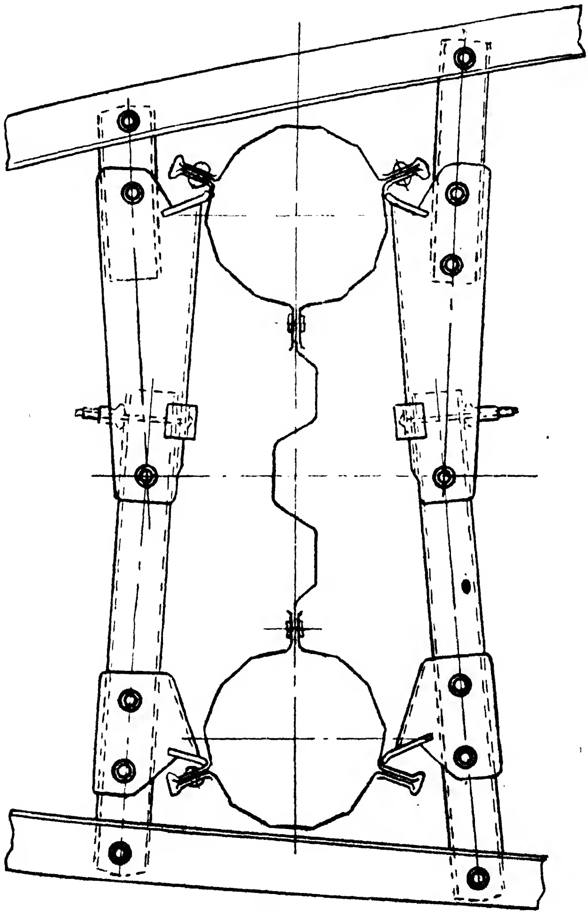


FIG. 77

The attachment consists of collapsible clamps, hinged at the centre and riveted at the outer end to the rib booms.

Small steel fillets attached to the clamps rest at the root of the spar lip. The rib is inserted with the clamps in the collapsed position, and they are then straightened out by tightening the cycle spoke nipple. shown projecting just above the hinge. This lengthens the fixing and causes the fillets to grip the spar booms.

Fig. 78 shows a spar of somewhat similar construction, the built-up booms being replaced by shallow tubular ones made in one piece. This has the advantage of being lighter, and the oval sections allow the bulk of the metal to be farther from the neutral axis.

It will be noticed that fittings are riveted to the booms. As this could not be done by the usual process at such a large distance from the open end, a special method is used. The rivet and process is illustrated in Fig. 79. The rivet is of mild steel and is the hollow type, having an external diameter of $\frac{1}{8}$ in. This is threaded on a high-tensile steel peg

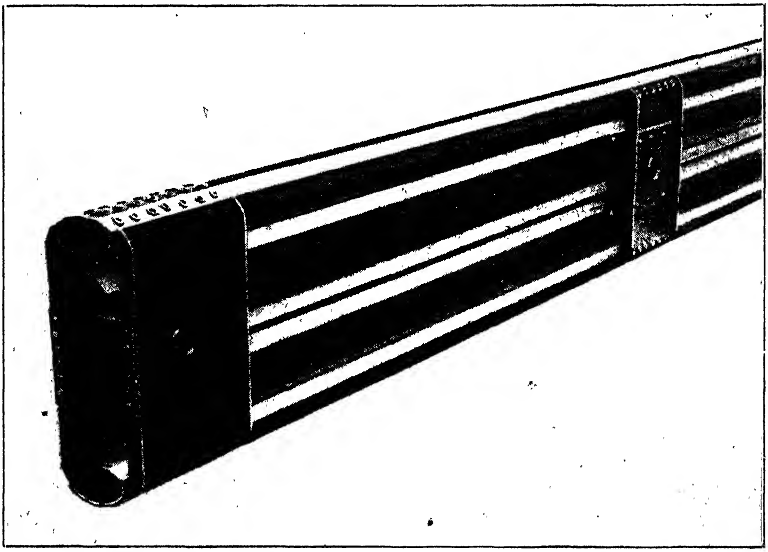


FIG. 78. ARMSTRONG SPAR WITH OVAL FLANGE

headed up to a cone shape at one end. The rivet and pin are then inserted into the hole, and the shank end of the peg gripped by the riveting tool, while the outside portion of the tool butts up against the flange of the rivet on the outside of the hole. The gripping part of the tool now draws the peg back through the rivet, whilst the outside still presses on the rivet, keeping it in place.

The first action of the peg is to expand the inside end of the rivet; then, as it meets with resistance at the part where the rivet is held in the hole, it is wire drawn to a smaller diameter, at the same time expanding the rivet tightly into the hole.

The Bristol type of spar and its attachment to the ribs is shown in Fig. 80 (*a*).

The simplicity of this design is worthy of note. It is built up of corrugated steel strip. The edges of the flanges and webs are grooved and form four similar interengaging edges, the spar being assembled by sliding the edges of the flanges along the grooves of the webs.

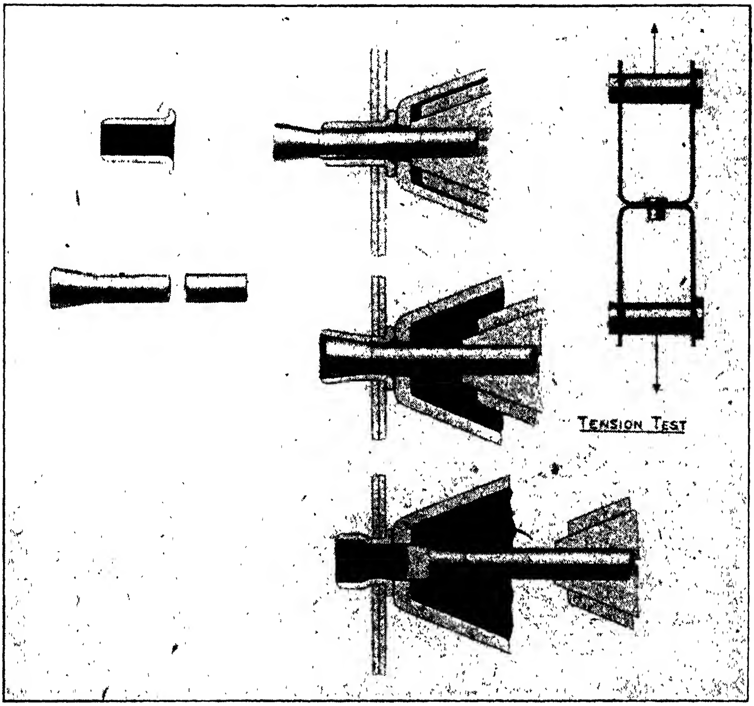


FIG. 79. METHOD OF CLOSING POP RIVETS ON ARMSTRONG SPARS

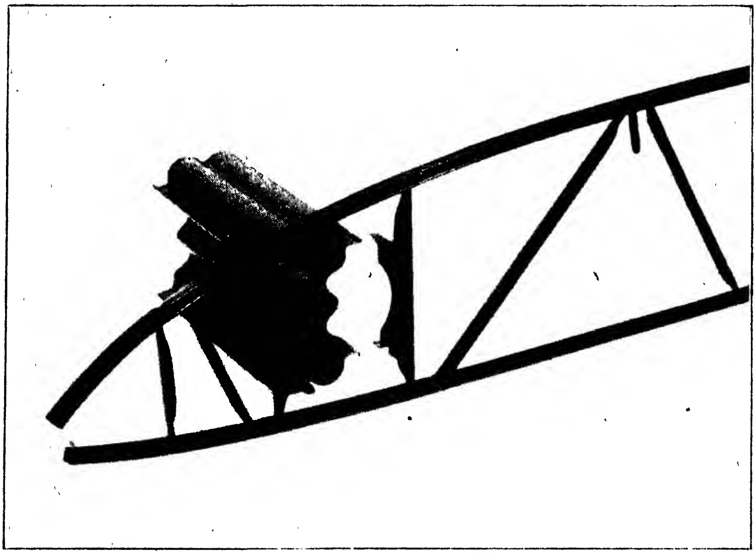


FIG. 80 (*a*). BRISTOL RIB ATTACHMENT

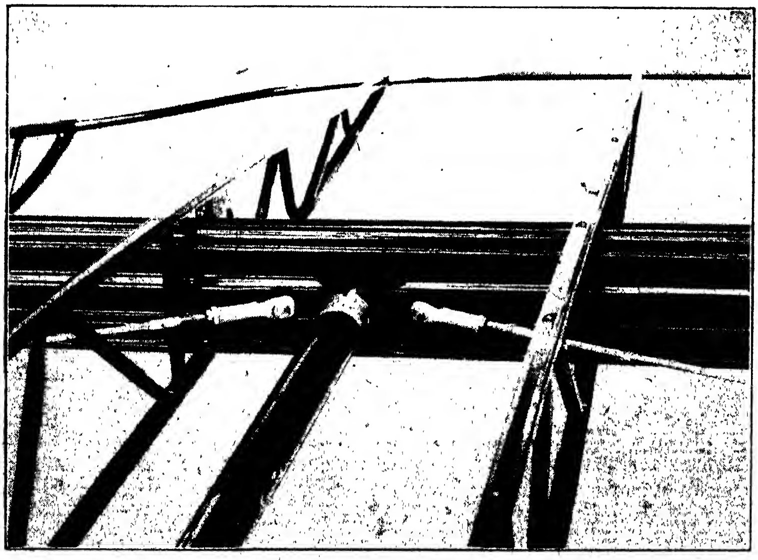
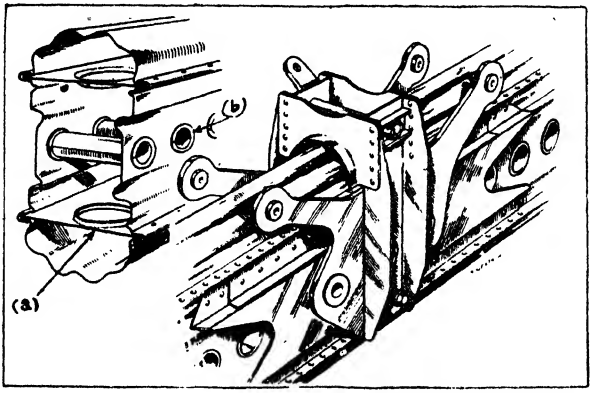


FIG. 80 (*b*). DRAG BRACING ATTACHMENT ON BRISTOL WING

No rivets are required, except those which secure the fittings and ribs. The ribs are secured by means of lugs on the rib posts, which fit between, and are riveted to, the spar tips.

Fig. 80 (*b*) shows how the drag bracing is attached. Side plates are



(*By courtesy of "Flight"*)

FIG. 81 (*a*)

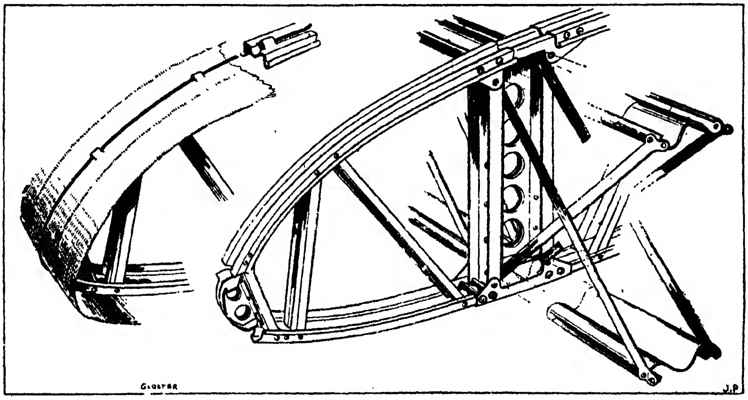


FIG. 81 (*b*) (*By courtesy of "Flight"*)

riveted to the tips on each side of the spar, an eyebolt is bolted through and tightened against a distance tube. This eyebolt acts as an anchorage for the wiring lugs and drag strut.

Fig. 81 (*a*) shows the Boulton & Paul spar and interplane bracing attachment on the *Sidestrand*. The tension members (*a*) are to prevent the flanges flattening out under load, and the object of the stiffener tubes (*b*) is to support the webs against secondary flexure and so prevent the spar

from collapsing prematurely through elastic instability of the webs, before the maximum flange stress is reached. At the same time they assist in the assembly of the spars, which is carried out as follows—

1. The two webs are punched and the distance tubes inserted and fixed by rolling over their outer edges.

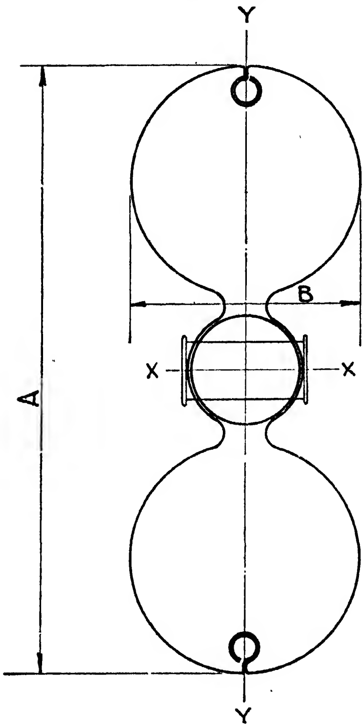


FIG. 81 (*c*)

2. The spar plates, which provide local stiffness at the points where fittings are attached, are riveted on to the webs.

3. Any other fittings which are permanently attached to the spars are assembled.

4. The two flanges are drawn into position along the grooves formed by the turning over of the edges of the webs, and, finally, the assembly of the spar is completed by the punching and riveting of the flanges to the webs.

Quite a different design is that due to the Gloster Company, illustrated in Fig. 81 (*b*). This type is specially suitable for thick wing sections. Good strength/weight ratio is obtained by taking full advantage of the depth of the section, and putting the bulk of the material at a distance from the neutral axis. It consists of two flanges, braced together by cross-bracing taking tension and posts taking compression. The shear will be taken by tension and compression in the bracing.

The ribs, which are in three portions—leading, training, and centre—are riveted to the spars at the compression posts. Note the special method of attaching fabric.

Fig. 81 (*c*) shows a Boulton & Paul two-piece spar for light planes. The two portions are joined, top and bottom, by small interlocking radii, which also help to make the flanges elastically stable. To make it more rigid, short tubes are put into the centre at intervals and secured by large diameter rivets.

A duralumin spar used in many Vickers machines is shown in Fig. 82. The elaborate corrugations used in steel spars are not necessary here, due to the comparative thickness of the metal used.

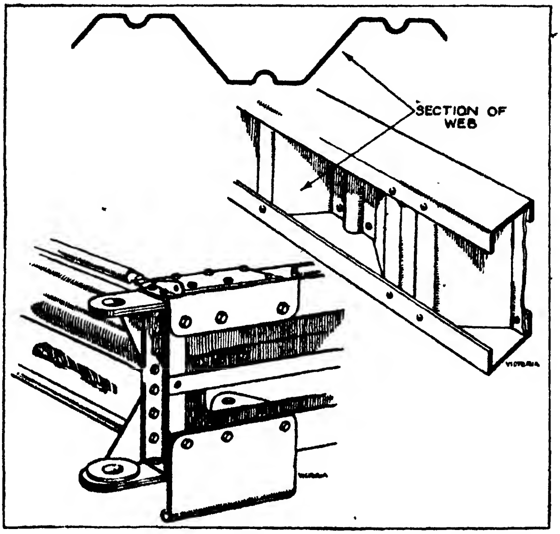
This spar consists of two flanges of channel section, connected together by a single duralumin strip, which at regular intervals crosses obliquely from one face of the spar to the other, and is riveted to the flanges, on one side or other, between each cross over.

Another interesting example of a duralumin spar is that used on

the Blackburn *Nile* flying boat. This, as will be seen from the illustration (Fig. 83), is very similar to that used for bridge construction.

Forming the Corrugations

In order to form the required section, metal strip of the developed width of the section is passed through consecutive sets of rollers, which



(*By courtesy of "Flight"*)

FIG. 82

bend the material past the yield point and so give it a permanent set; each roller in turn shaping the material more, until the required shape is reached.

If high-tensile steel is used, there will be a considerable "spring-back," i.e. the material on coming out of the rolls will return partly to its original shape.

The last roll must therefore bend the material past the required amount, so that on release it springs back to the correct shape.

In the drawing and rolling of steel sections there are two schools of thought over the question of heat treatment. In some cases the strip is received heat treated to its final strength, and rolled cold in this hard condition. In others it is bought in the annealed condition and rolled or drawn in this state, afterwards being passed through a die of the final shape during heat treatment.

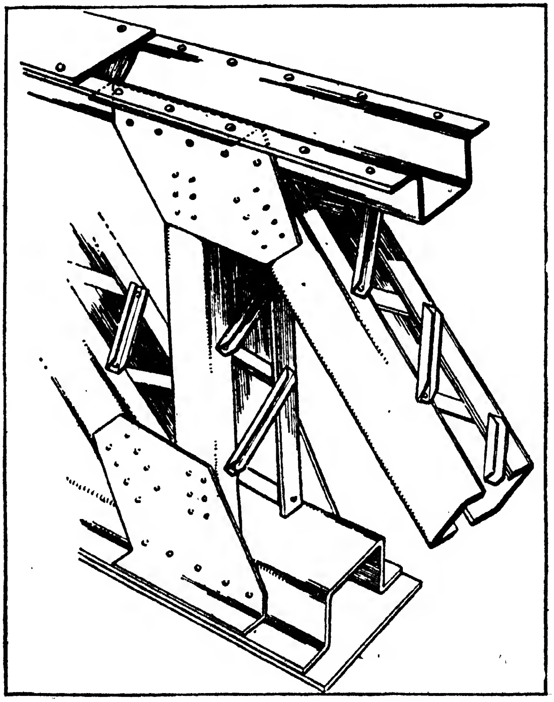
Hard rolling reduces the number of operations, and soft rolling reduces wear of rolls and dies, and difficulties of spring-back are eliminated.

With a section such as Fig. 87 (4), the material must be shaped by drawing through a die, or by making the top roll in three parts (*a*)

Fig. 88 (a). as the distance x being less than y, it would be impossible for the pair of rolls to run in each other.

The whole process may be done by drawing, but this is normally a longer job, the material being liable to twist if drawn through more than one die at a time.

The rolls and dies must be made very accurately, so that the material is not gripped unevenly. Allow a lead in on the dies; make, say, $\frac{1}{4}$ in.



(*By courtesy of "Flight"*)

Fig. 83

the true section and chamfer the rest off. The axes of the rolls should be equidistant from the centre line of the strip, in order to minimize the amount of slip caused by the difference in linear velocity of the rolls, which increases with the difference of diameter at any point. Failure to observe any of the above may lead to the section coming out twisted.

Fig. 84 shows the large and small rolling mill at the Bristol works. The strip may be seen going through the first pair of rolls at (a). The drawing process is illustrated in Fig. 85.

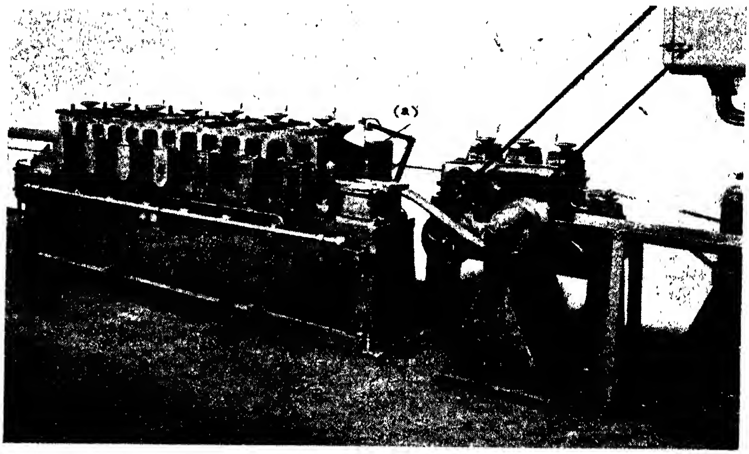


FIG. 84. ROLLING MILLS FOR METAL STRIP AT THE BRISTOL WORKS

FIG. 85. DRAW BENCHES AT THE BRISTOL WORKS

Design of Rolls and Dies

The first rolls are got out by sight from experience, the controlling factor being that the material must be shaped in easy stages. This also decides the number of stages; usually four or five are sufficient for a spar section.

The last roll but one is usually made of similar dimensions to the finished section. The last has to be such that the permanent set will be

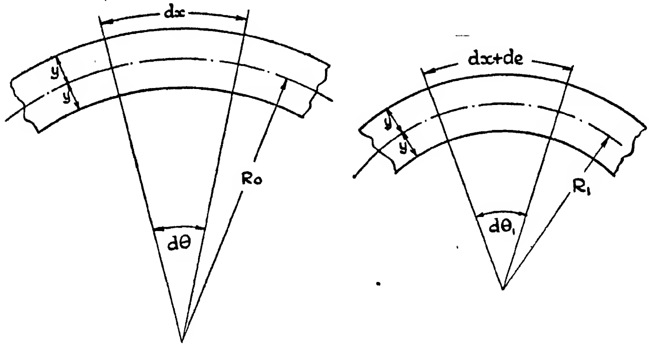


FIG. 86 (*a*)

the correct section, i.e. it will be bent further than the finished product, there being a considerable spring-back if high tensile steel is used.

The shape is arrived at in the following manner—

Consider a length dx of the cross-section of a bent strip of radius R_0, and let the radius of the die be R_1, as shown in Fig. 86 (*a*). Let the stress in the die be f. It will not be subjected to stress when out of the die.

Let the increase in length of dx in the die be de.

Then $dx = (R_0 + y)d\theta$

$$de = (R_1 + y)d\theta_1 - (R_0 + y)d\theta$$
$$= (R_1 + y)(d\theta \,.\, R_0/R_1) - (R_0 + y)d\theta \quad (de \text{ and } y \text{ being small to } R)$$

$$\frac{f}{E} = \text{Strain} = \frac{de}{dx} = \frac{(R_1 + y)(d\theta \,.\, R_0/R_1) - (R_0 + y)d\theta}{(R_0 + y)d\theta}$$

$$\frac{f}{E} = \frac{R_0 + y\,(R_0/R_1) - R_0 - y}{R_0 + y}$$
$$= y(R_0 - R_1)/R_1R_0 \quad (y \text{ being small})$$
$$= y\left(\frac{1}{R_1} - \frac{1}{R_0}\right)$$

y will equal half-thickness, and f will equal yield stress.

If there is no marked yield stress, a stress slightly greater than 0·1 per cent proof stress will give an approximate result.

Note stresses used should be taken from test, and not from specifications.

An example will make the method clear. Design the necessary rolls and dies for the spar flange shown in Fig. 86 (*b*).

It is obviously necessary that all corresponding arcs should be of the same length, otherwise part of the strip would be bent up in one roll

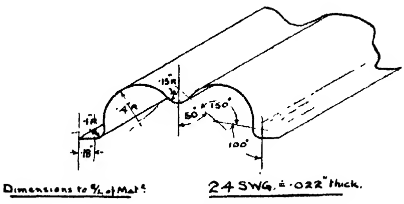


FIG. 86 (*b*)

and down in the next, e.g. the arc of 1·1 in. radius in Fig. 87 (1) must be equal in length to arcs of 0·6 in. radius (2), 0·4 in. radius (3), and 0·337 in. radius (4).

Denote the arc of 0·15 in. radius (Fig. 86 (*b*)) by a, 0·4 in. radius by b, and 0·1 in. radius by c.

In order that the first stage may be easy, we will make radius a (R_a) 0·4 in., (R_b) 1·1 in., and (R_c) 0·3 in.

The corresponding angles will be found in the following manner—

$$\text{Angle for equal arc} = \frac{\text{final radius} \times \text{final angle}}{\text{new radius}}$$

$$\frac{\text{Angle of } a}{2} = \frac{0{\cdot}15 \times 50}{0{\cdot}4} = 18\tfrac{3}{4}^\circ.$$

$$\text{Angle of } b = \frac{0{\cdot}4 \times 150}{1{\cdot}1} = 54\tfrac{1}{2}^\circ.$$

$$\text{Angle of } c = \frac{0{\cdot}1 \times 100}{0{\cdot}3} = 33\tfrac{1}{3}^\circ.$$

Roll 2 is found in the same manner as Roll 1. Roll 3 is made the same as the final flange, and 4, which must be a die (x being less than y), is found from the formula given above, as follows—

Assume the yield stress is 70 tons/sq. in. and E = 13600 tons/sq. in. for this example.

Then
$$\frac{1}{R_{a4}} - \frac{1}{R_{a3}} = \frac{f_2}{Ey}$$

$$\frac{1}{R_{a4}} = \frac{f_2}{Ey} + \frac{1}{R_{a3}}$$

$$= \frac{70}{13600 \times 0{\cdot}011} + \frac{1}{0{\cdot}15}$$

$$= \frac{1}{2{\cdot}137} + \frac{1}{0{\cdot}15}$$

$$= \frac{1}{\underline{0{\cdot}14}}.$$

Radius a of die 4 $= R_{a4} = \underline{\underline{0{\cdot}14 \text{ in.}}}$

$$\frac{1}{R_{b4}} = \frac{1}{2{\cdot}137} + \frac{1}{0{\cdot}4}$$

$$= \frac{1}{0{\cdot}337}$$

$$R_{b4} = \underline{\underline{0{\cdot}337 \text{ in.}}}$$

$$\frac{1}{R_{c4}} = \frac{1}{2{\cdot}137} + \frac{1}{0{\cdot}1}$$

$$= \frac{1}{0{\cdot}096}$$

$$R_{c4} = \underline{\underline{0{\cdot}096 \text{ in.}}}$$

The angles found as for Roll 1 are—

a	53·7°
b	178·7°
c	104·3°

All results (Fig. 87) are given to the centre line of the strip, and so the actual rolls and die will have half the thickness (0·011 in.) added or subtracted in order to allow for the material.

Rolls 3 are given as an example in Fig. 88 (b).

THE RIBS

The ribs are frames usually of either the "Warren" or "N" girder type. They need careful design; there being many of them, much weight can be saved by getting the best distribution of the bracing. The bracing members which take the greatest loads are next to the spars and should be diagonally upwards, as in this way they will take a tensile load. Members taking large compression loads should be short at the expense of tension members and those taking smaller compressions.

Fig. 89 is a line drawing of a rib, showing how the loads are distributed, and how the bracing has been arranged to suit.

The Bristol type shown in Figs. 49 and 80 (a) is of high-grade steel, of channel section. The diagonal members are continuous between the spars, being folded flat where riveted to the booms. This allows of the quick assembly illustrated in Fig. 90.

The majority of braced ribs use some form of channel, while others are tubular members.

Quite another type of rib is a one-piece pressing. This is very suitable for mass production, being cheap and easy to make after the necessary press tools have been produced. An illustration of this type, as used in the Avro *Trainer*, is shown in Fig. 91, it has the advantage of being strong in torsion.

The *Blenheim* rib, which is an Alclad pressing, is shown in Fig. 50, and Fig. 50 (b) shows a similar type of rib used on the *Battle*. These ribs are particularly suitable for attaching the metal skin, and also have the torsional stiffness required for modern high speeds.

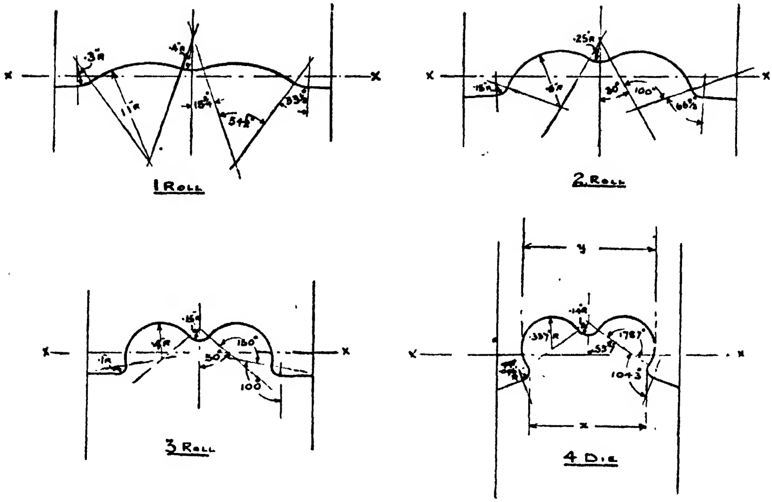


Fig. 87

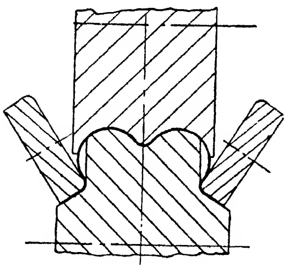


Fig. 88 (*a*)

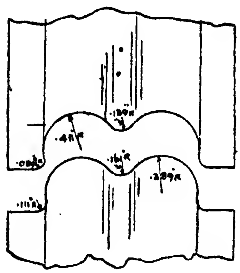


Fig. 88 (*b*)

Fittings

The fittings at the different joints in a braced structure account for a large proportion of the weight of the machine.

Not only must they be considered from the strength/weight standpoint, but care must be taken that the joint is made in such a way that no extra loads are put on the structure.

Referring to the fuselage joint (Fig. 92 (*a*)), the line of action of the wire being offset from the point where the line of action of the strut and longeron meet, a bending moment Px will be applied to the longeron, thus increasing the stress in this member. In the same way, in the side view (*b*), the longeron will have a torsion Ry. This will be overcome if the wires are arranged as shown by the dotted lines. Fig. 92 (*c*) is a wing joint of Boulton & Paul design, showing how the lines of action

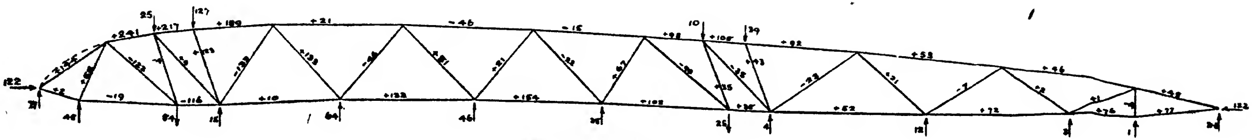


FIG. 89

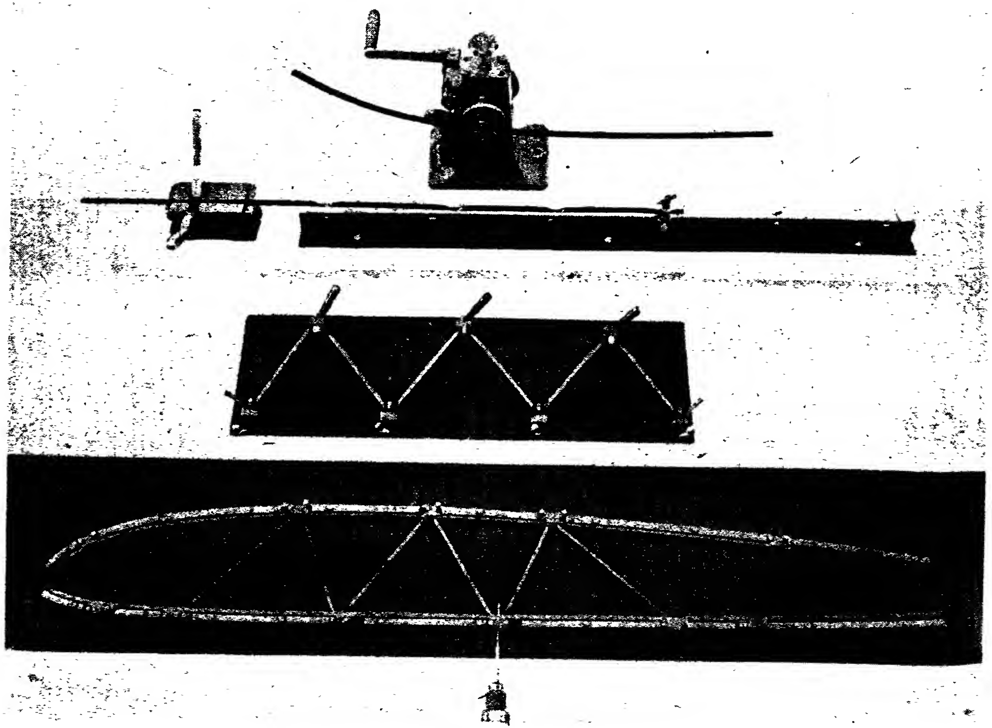


FIG. 90. METHOD OF RIB ASSEMBLY (BRISTOL)

of all the loads are arranged to meet at a point. Sometimes on spars offsets are used to give a moment to counteract the bending moment. This was done on the wing Fig. 49, where the root-end eyebolts are above the spar centre line.

A socket on a tube end is chamfered down to about $\frac{1}{64}$ in., in order that there will be no sudden decrease in sectional area.

In some cases tubes are flattened for purposes of attachment, in which case a liner tube of less thickness than the main tube, and having a

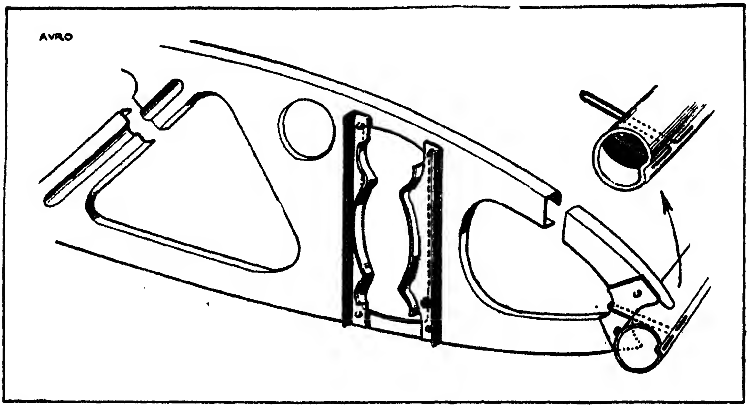


FIG. 91

serrated end, must be inserted. This, as with the socket, is to prevent failure, due to sudden change of area.

It is often necessary, where a bolt bears on thin sheet metal, to give it a larger diameter than is required for the load in the bolt. This is in order that the bearing stress may not be too great. The bolt, however, need not be appreciably heavier, as it can be lightened by drilling a hole up the centre.

Wiring Lugs

The loads from the bracing wires are usually first taken on a sheet metal lug, which may be an integral part of the main fitting, or a separate detail attached to the fitting.

A lug may fail in three ways—

1. It may tear along the line of maximum stress.
2. It may break away in front of the pin.
3. It may crush in front of the pin, due to the bearing area being too small.

The size of wire will determine the fork end to be used, and as these are standardized, the size of pin and pin hole is fixed.

The thickness must be such that

$$dt = \frac{P}{f_b}$$

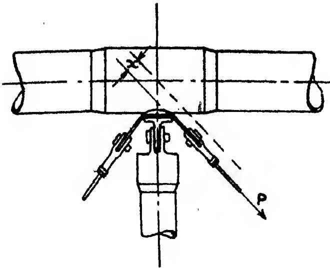


FIG. 92 (*a*)

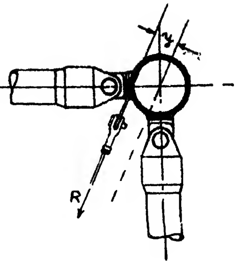
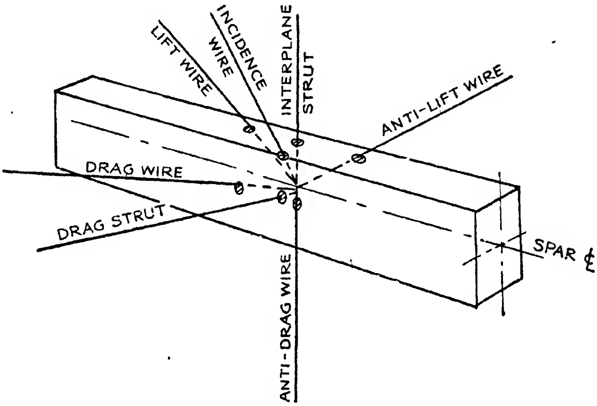


FIG. 92 (*b*)



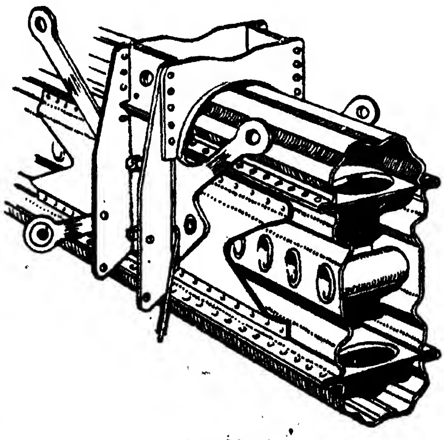


FIG. 92 (*c*)

where d = diameter of pin
t = thickness of lug
P = load in wire
f_b = permissible bearing stress.

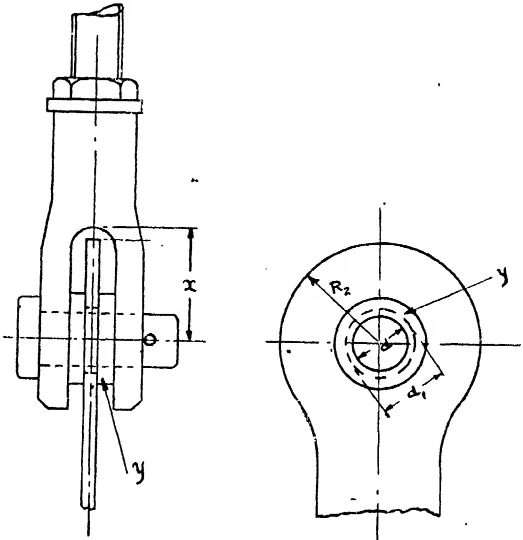


FIG. 93

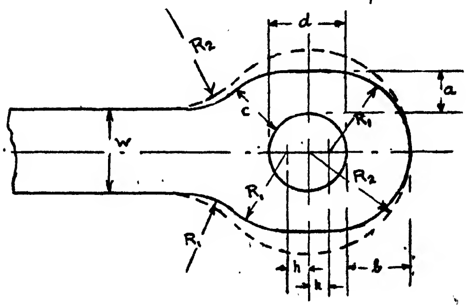


FIG. 94

This thickness must also be such that R_2 is less than x (Fig. 93), so as to leave a clearance between the lug and fork end.

Where high-tensile steel wiring lugs are used, it may be found necessary to fit a sleeve in the lug (Fig. 93 (y)). This prevents a large bending stress being put on the pin due to the thinness of the lug, and the width of the standard fork-ends.

It also reduces the bearing stress by the ratio d/d_1.

There is some doubt about the distribution of stress in a lug, and the proportions are usually obtained from formula derived from tests.

The "offset" lug shown in Fig. 94 is of practically uniform strength; many designers, however, use the concentric one, shown dotted.

Suitable formulae for the design of lugs of either type are given below—

$$W = \frac{P}{tf}$$

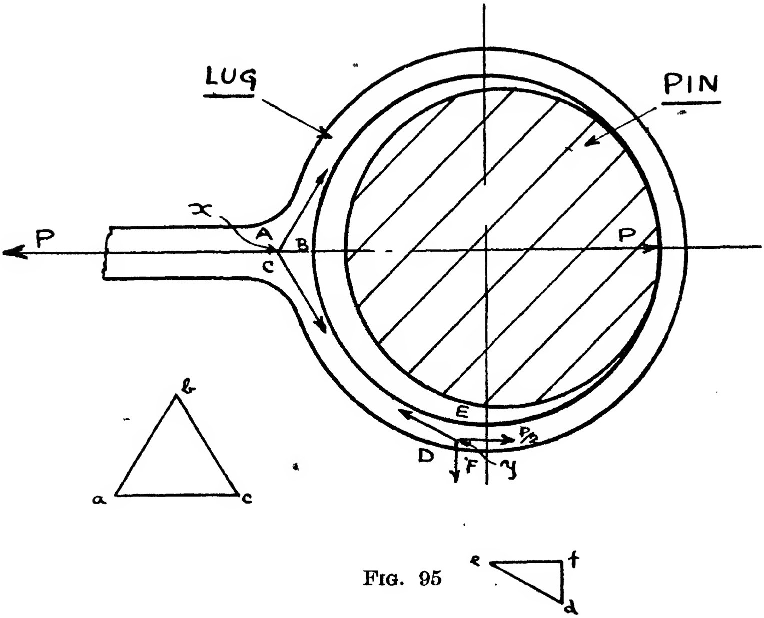


FIG. 95

where t = thickness

f = ultimate strength

$$a = 0{\cdot}555W$$

$$R_1 = 0{\cdot}555W + \frac{d}{2}$$

$$b = 1{\cdot}25a$$

$$R_2 = 0{\cdot}695W + \frac{d}{2}$$

$$h = R_2 - R_1 = 0{\cdot}14W.$$

The reason why c is greater than a and why a is greater than $\frac{W}{2}$ may be understood by considering an exaggerated lug (Fig. 95). By drawing the triangle of forces for the forces at the point x, it will be seen that

the forces AB and BC are greater than half the force AC, due to their angularity.

Again, considering the triangle of forces for the point y, there must

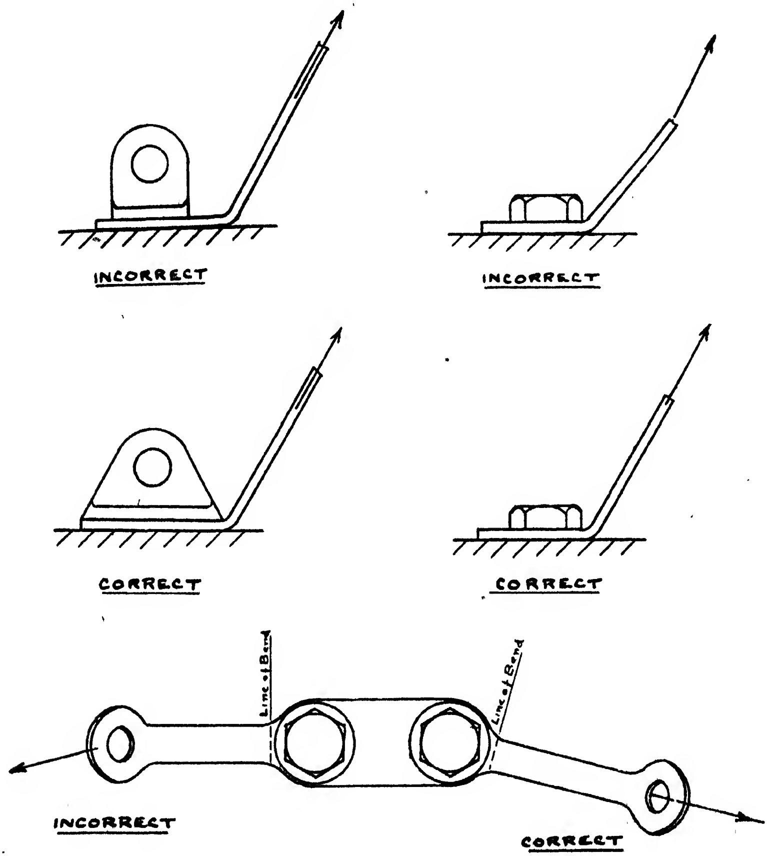


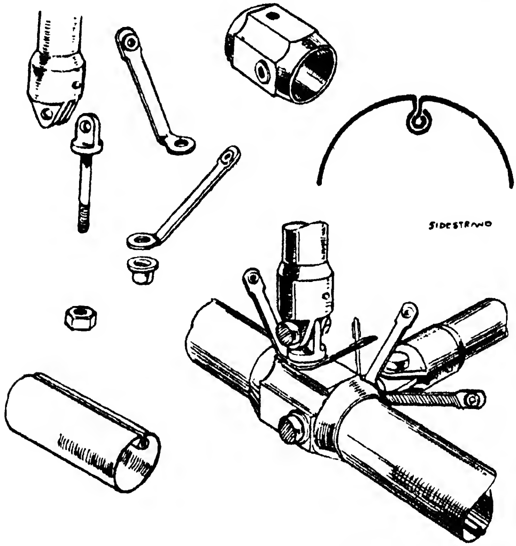
FIG. 96

be a force FD exerted by the material resisting the tendency for the lug to bend inwards at this point. The lug must therefore withstand bending as well as direct load at y.

Lugs must always be bent to the angle of the wire which they serve. The bend must be close to the bolt, or other holding-down member, otherwise they will straighten out and put the machine out of alignment. In order that the lug may not tend to tear, it must be in line with

the wire, i.e. its line of bend must be at right angles to the line of action of the wire.

Fig. 96 shows correct and incorrect ways of bending lugs.

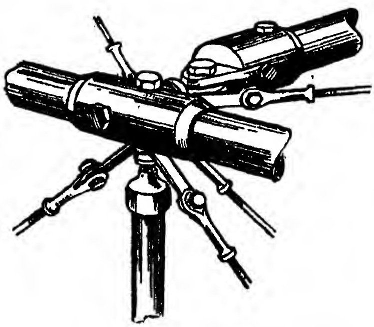


(*By courtesy of "Flight"*)

FIG. 97

Types of Fittings

Fittings are so many and varied that it is quite impossible in one chapter to do more than illustrate a few of the widely differing types.



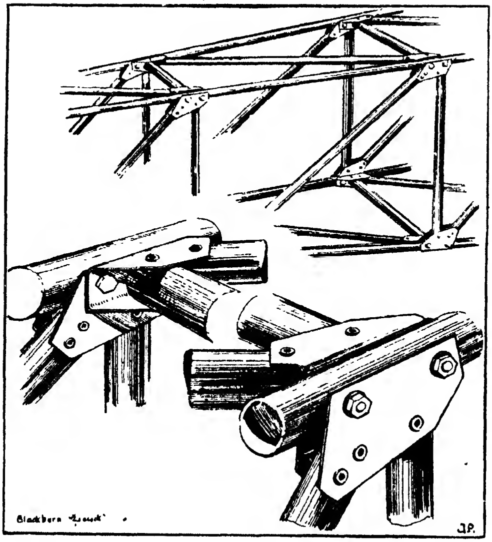
(*By courtesy of "Flight"*)

FIG. 98

(*By courtesy of "Flight"*)

FIG. 99

Fig. 97 shows the details and assembly of the fuselage joint in the Boulton & Paul *Sidestrand*. The longerons are of "closed joint" tube,



(*By courtesy of "Flight"*)

FIG. 100

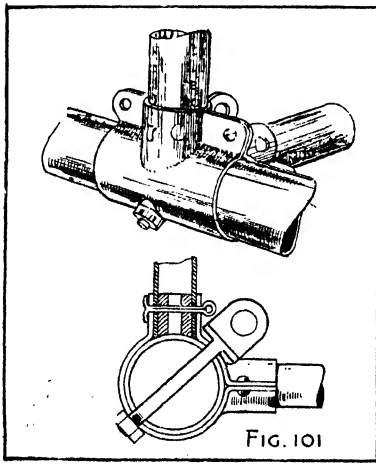


FIG. 101

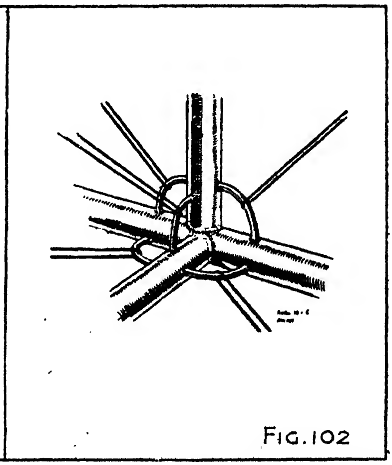


FIG. 102

(*By courtesy of "Flight"*)

made by drawing steel strip through a series of dies. A sleeve having four flat surfaces is fitted to the longeron at the joint, and this is drilled with the longerons to take two eyebolts at right angles, and slightly staggered so that their shanks will clear each other. The lugs fit under the heads of the bolts, and the horizontal and vertical bracing struts have socket fork-ends, by which they are pinned to the eyebolts.

A very similar type of joint is used in the Armstrong *Atlas* (Fig. 98). The sleeve being round, a shaped washer must be fitted between the bolts and the sleeve to give a flat surface for tightening up against.

The vertical strut has a socket with a "cup," into which fits the spherical head of the bolt through the longeron; the whole being kept in position by the tension in the wires. An advantage of the ball-joint is that no initial bending can be put on the strut due to misalignment, etc.

Fig. 99 shows a fuselage joint by the Bristol Company. Here the longerons and struts are made of two pieces of rolled steel strip, assembled in the same way as the Bristol spars, previously described. The extreme edges of the struts are cut away, an inch or so from the end, and a flat plate is inserted and riveted between the two portions. The other end of the plate is riveted to the lip of the longeron. The only wire bracings are the bulkhead wires, and these are taken by a small eyepiece, bolted to the vertical and horizontal plates. This makes a very simple and light joint, and has the advantage that if a thin, large, diameter strut is used, the save in weight thereby obtained is not lost by the necessity of a larger and heavier end fitting.

Another type using plates, round steel tubes, and no wires is the Blackburn *Lincock* (Fig. 100). Here the plates are bolted to the longerons, and the struts are attached to them by tubular rivets.

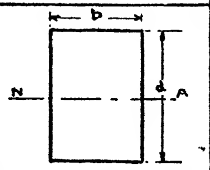
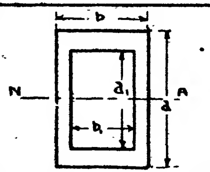
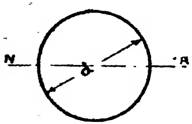
A type used by Gloster's is shown in Fig. 101. A steel plate is wrapped round the longeron to form lugs for the side wires, and sockets for the struts. The bulkhead wires are taken by an eyebolt, bolted through the wrapper plate.

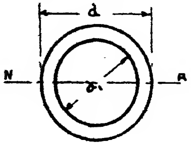
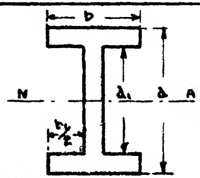
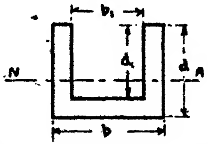
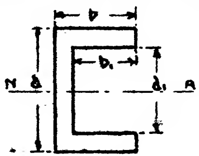
A welded fuselage joint as used on the *Avian* is illustrated in Fig. 102. At present welded construction is not very popular in this country, though it is used extensively on the Continent. This is probably due to the British constructors' preference for the high-grade steels, which are not suitable for welding.

Examples of joints used in stressed-skin construction will be seen in Chapter VI.

APPENDIX I

Some Useful Constants of Standard Sections

SECTION	AREA A	Distance of Outermost Fibre from Neutral Axis y	Moment of Inertia about Neutral Axis I_{NA}	Modulus of Section $Z = \frac{I_{NA}}{y}$	Radius of Gyration $K = \sqrt{\frac{I_{NA}}{A}}$
	bd	$\frac{d}{2}$	$\frac{1}{12}\ bd^3$	$\frac{1}{6}\ bd^2$	$\frac{d}{\sqrt{12}}$
	$bd - b_1d_1$	$\frac{d}{2}$	$\frac{bd^3 - b_1d_1^3}{12}$	$\frac{bd^3 - b_1d_1^3}{6d}$	$\sqrt{\frac{bd^3 - b_1d_1^3}{12(bd - b_1d_1)}}$
	$\frac{\pi}{4}\ d^2$	$\frac{d}{2}$	$\frac{\pi d^4}{64}$	$\frac{\pi d^3}{32}$	$\frac{d}{4}$

	$\frac{\pi}{4}(d^2-d_1^2)$	$\frac{d}{2}$	$\frac{\pi}{64}\ (d^4-d_1^4)$	$\frac{\pi}{32}\ \frac{(d^4-d_1^4)}{d}$	$\sqrt{\frac{d^2+d_1^2}{4}}$
	$bd-b_1d_1$	$\frac{d}{2}$	$\frac{bd^3-b_1d_1^3}{12}$	$\frac{bd^3-b_1d_1^3}{6d}$	$\sqrt{\frac{bd^3-b_1d_1^3}{12(bd-b_1d_1)}}$
	$bd-b_1d_1$	$\frac{bd^2-b_1d_1^2}{2(bd-b_1d_1)}$	$\frac{bd^3-b_1d_1^3}{3}-\frac{(bd^2-b_1d_1^2)^2}{4(bd-b_1d_1)}$	$\frac{2(bd-b_1d_1)(bd^3-b_1d_1^3)}{3(bd^2-b_1d_1^2)}-\frac{1}{2}(bd^2-b_1d_1^2)$	$\sqrt{\frac{bd^3-b_1d_1^3}{3(bd-b_1d_1)}-\left(\frac{bd^2-b_1d_1^2}{2(bd-b_1d_1)}\right)^2}$
	$bd-b_1d_1$	$\frac{d}{2}$	$\frac{bd^3-b_1d_1^3}{12}$	$\frac{bd^3-b_1d_1^3}{6d}$	$\sqrt{\frac{bd^3-b_1d_1^3}{12(bd-b_1d_1)}}$

APPENDIX II

MOMENT OF INERTIA OF ROLLED METAL SECTIONS

Now that rolled metal sections are so much in use, it is desirable that their Moment of Inertia may be found by others than those familiar with the use of calculus.

Below are given formulae by the use of which, it is hoped, readers of this book may find the Moment of Inertia of any thin metal section. As they are derived by the use of calculus, no proof will be given.

Let AB (Fig. 103 (a)) be a portion of arc of radius r to the centre, and thickness t.

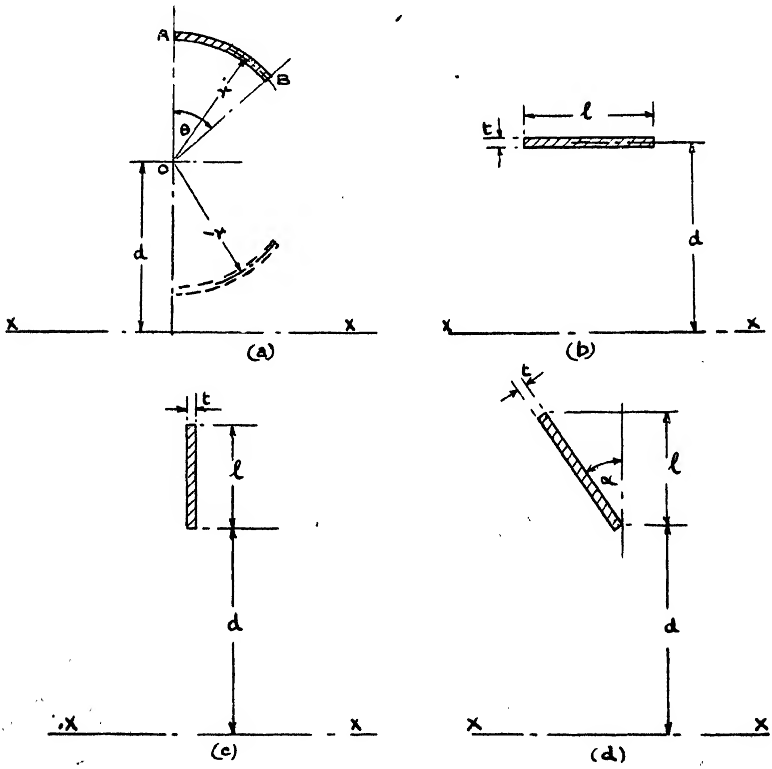


FIG. 103

AO is at right angles to the axis xx about which the Moment of Inertia is required.

$$\text{Then } I_{xx} = tr\left[\left(d^2 + \frac{r^2}{2}\right)\theta + 2dr\sin\theta + \frac{r^2}{4}\sin 2\theta\right]$$

Note. θ will be in radians $= \dfrac{\text{degrees}}{57{\cdot}3}$.

If the arc is nearer xx than o (as shown dotted) ($2dr \sin\theta$) will be negative.

For a flat portion parallel to (xx) (Fig. 103 (b)),

$$I_{xx} = tld^2.$$

For a flat portion at right angles to (xx) (Fig. 103 (c)),

$$I_{xx} = t\left[\frac{(d+l)^3}{3} - \frac{d^3}{3}\right]$$

For a flat diagonal portion (Fig. 103 (d)),

$$I_{xx} = \frac{t}{\cos a}\left[\frac{(d+l)^3}{3} - \frac{d^3}{3}\right]$$

By dividing the section into these portions and adding the results, the Moment of Inertia of the whole section may be found.

EXAMPLE 1. Find I_{xx} for the section shown in Fig. 104. Dimensions given to centre of material.

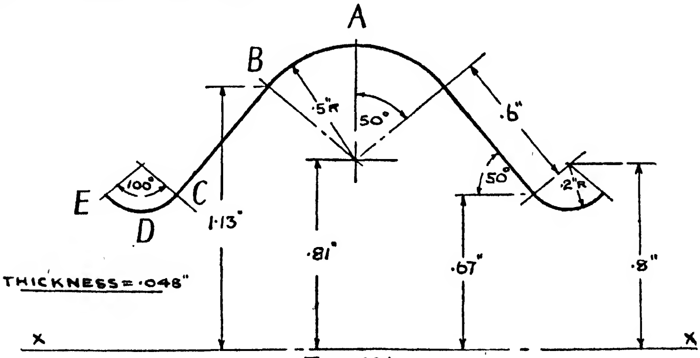


FIG. 104

$$I \text{ of arc } AB = tr\left[\left(d^2 + \frac{r^2}{2}\right)\theta + 2dr\sin\theta + \frac{r^2}{4}\sin 2\theta\right]$$

$$= 0{\cdot}048 \times 0{\cdot}5\left[\left(0{\cdot}81^2 + \frac{0{\cdot}5^2}{2}\right)\frac{50}{57{\cdot}3} + 2 \times 0{\cdot}81 \times 0{\cdot}5 \sin 50 + \frac{0{\cdot}5^2}{4}\sin 100\right]$$

$$= 0{\cdot}024\,[(0{\cdot}6561 + 0{\cdot}125)\,0{\cdot}8727 + 2 \times 0{\cdot}81 \times 0{\cdot}5 \times 0{\cdot}766 + 0{\cdot}0625 \times 0{\cdot}9848]$$

$$= 0{\cdot}024\,(0{\cdot}6817 + 0{\cdot}6204 + 0{\cdot}0615)$$

$$= 0{\cdot}024 \times 1{\cdot}3636$$

$$= \underline{0{\cdot}0327} \text{ in.}^4$$

$$I \text{ of diagonal } BC = \frac{t}{\cos a}\left[\frac{(d+l)^3}{3} - \frac{d^3}{3}\right]$$
$$= \frac{0{\cdot}048}{\cos 40}\left[\frac{1{\cdot}13^3}{3} - \frac{0{\cdot}67^3}{3}\right]$$
$$= \frac{0{\cdot}048}{0{\cdot}766}\left[\frac{1{\cdot}4429}{3} - \frac{0{\cdot}3007}{3}\right]$$
$$= \frac{0{\cdot}048}{0{\cdot}766} \times 0{\cdot}3807$$
$$= \underline{\underline{0{\cdot}0238 \text{ in.}^4}}$$

$$I \text{ for arc } CD = tr\left[\left(d^2 + \frac{r^2}{2}\right)\theta - 2dr\sin\theta + \frac{r^2}{4}\sin 2\theta\right]$$
$$= 0{\cdot}048 \times 0{\cdot}2\left[\left(0{\cdot}8^2 + \frac{0{\cdot}2^2}{2}\right)\frac{50}{57{\cdot}3} - 2 \times 0{\cdot}8 \times 0{\cdot}2 \sin 50 + \frac{0{\cdot}2^2}{4}\sin 100\right]$$
$$= 0{\cdot}0096\,[(0{\cdot}64 + 0{\cdot}02)\,0{\cdot}8727 - 2 \times 0{\cdot}8 \times 0{\cdot}2 \times 0{\cdot}766 + 0{\cdot}01 \times 0{\cdot}9848]$$
$$= 0{\cdot}0096\,(0{\cdot}576 - 0{\cdot}2451 + 0{\cdot}0098)$$
$$= 0{\cdot}0096 \times 0{\cdot}3407$$
$$= \underline{\underline{0{\cdot}0032 \text{ in.}^4}}$$

I for arc DE will be the same as for CD, i.e. $= 0{\cdot}0032$ in.^{4}

Total I for half section AE

$$= 0{\cdot}0327 + 0{\cdot}0238 + 0{\cdot}0032 + 0{\cdot}0032$$
$$= \underline{\underline{0{\cdot}0629 \text{ in.}^4}}$$

$$I \text{ of whole section} = 2 \times 0{\cdot}0629$$
$$= \underline{\underline{0{\cdot}1258 \text{ in.}^4}}$$

EXAMPLE 2. Find the Moment of Inertia of the spar shown in Fig. 105, about the neutral axis.

Thickness of flanges = 0·022 in., thickness of webs = 0·018 in.

By drawing the section ten times full size and scaling, the centre distances may be found.

$$I \text{ arc } AB = 0{\cdot}022 \times 0{\cdot}15\left[\left(1{\cdot}518^2 + \frac{0{\cdot}15^2}{2}\right)\frac{50}{57{\cdot}3} - 2 \times 1{\cdot}518 \times 0{\cdot}15 \sin 50 + \frac{0{\cdot}15^2}{4}\sin 100\right]$$
$$= \underline{\underline{0{\cdot}0055}}$$

$$I \text{ arc } BC = 0{\cdot}022 \times 0{\cdot}4\left[\left(1{\cdot}167^2 + \frac{0{\cdot}4^2}{2}\right)\frac{50}{57{\cdot}3} + 2 \times 1{\cdot}167 \times 0{\cdot}4 \sin 50 + \frac{0{\cdot}4^2}{4}\sin 100\right]$$
$$= \underline{\underline{0{\cdot}0168}}$$

$$I \text{ arc } CD = 0{\cdot}022 \times 0{\cdot}4\left[\left(1{\cdot}167^2 + \frac{0{\cdot}4^2}{2}\right)\frac{100}{57{\cdot}3} + 2 \times 1{\cdot}167 \times 0{\cdot}4 \sin 100 + \frac{0{\cdot}4^2}{4}\sin 200\right]$$

$$= \underline{\underline{0{\cdot}0301}}$$

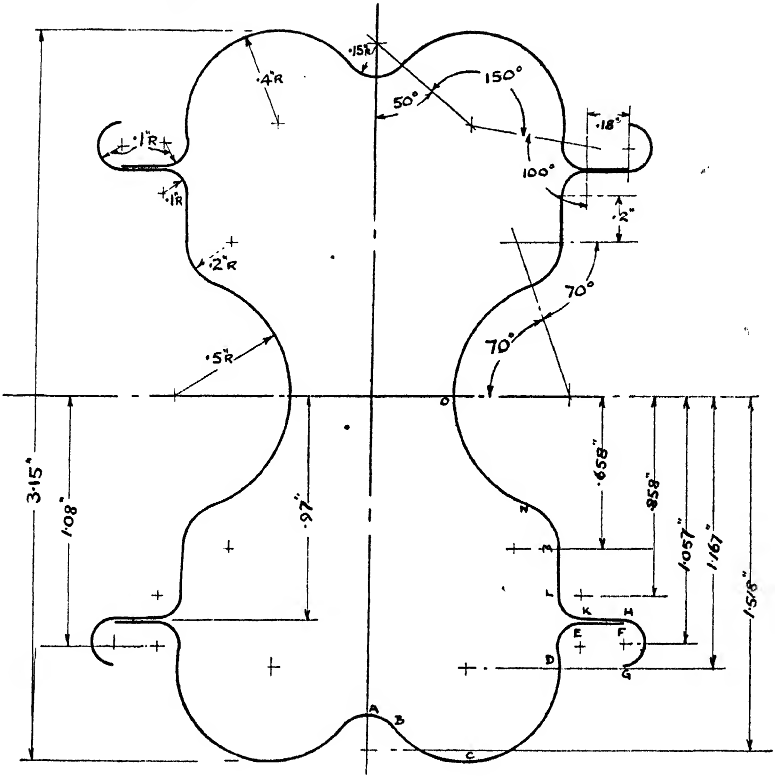


FIG. 105

$$I \text{ arc } DE = 0{\cdot}022 \times 0{\cdot}1\left[\left(1{\cdot}08^2 + \frac{0{\cdot}1^2}{2}\right)\frac{100}{57{\cdot}3} - 2 \times 1{\cdot}08 \times 0{\cdot}1 \sin 100 \, \frac{0{\cdot}1^2}{4}\sin 200\right]$$

$$= \underline{\underline{0{\cdot}0030}}$$

$$I.EF = 0{\cdot}022 \times 0{\cdot}18 \times 0{\cdot}981^2$$
$$= \underline{\underline{0{\cdot}0038}}$$

$$I \text{ of half flange} = 0{\cdot}0055 + 0{\cdot}0168 + 0{\cdot}0301 + 0{\cdot}0030 + 0{\cdot}0038$$
$$= \underline{\underline{0{\cdot}0592}}$$

$$I \text{ of the two flanges} = 0{\cdot}0592 \times 4$$
$$= \underline{\underline{0{\cdot}2368 \text{ in.}^4}}$$

$$I \text{ arc } GH = 0{\cdot}018 \times 0{\cdot}1 \left[\left(1{\cdot}057^2 + \frac{0{\cdot}1^2}{2}\right)\frac{180}{57{\cdot}3}\right]$$
$$= \underline{\underline{0{\cdot}0060}}$$

$$I.HK = 0{\cdot}018 \times 0{\cdot}18 \times 0{\cdot}961^2$$
$$= \underline{\underline{0{\cdot}0030}}$$

$$I \text{ arc } KL = 0{\cdot}018 \times 0{\cdot}1\left[\left(0{\cdot}858^2 + \frac{0{\cdot}1^2}{2}\right)\frac{90}{57{\cdot}3} + 2 \times 0{\cdot}858 \times 0{\cdot}1 \sin 90\right]$$
$$= \underline{\underline{0{\cdot}0024}}$$

$$I.LM = 0{\cdot}018\left[\frac{0{\cdot}858^3}{3} - \frac{0{\cdot}658^3}{3}\right]$$
$$= \underline{\underline{0{\cdot}0021}}$$

$$I \text{ arc } MN = 0{\cdot}018 \times 0{\cdot}2\left[\left(0{\cdot}658^2 + \frac{0{\cdot}2^2}{2}\right)\frac{90}{57{\cdot}3} - 2 \times 0{\cdot}658 \times 0{\cdot}2 \sin 90\right]$$
$$= 0{\cdot}018 \times 0{\cdot}2\left[\left(0{\cdot}658^2 + \frac{0{\cdot}2^2}{2}\right)\frac{20}{57{\cdot}3} - 2 \times 0{\cdot}658 \times 0{\cdot}2 \sin 20 + \frac{0{\cdot}2^2}{4}\sin 40\right]$$
$$= 0{\cdot}018 \times 0{\cdot}2\left[\left(0{\cdot}658 + \frac{0{\cdot}2^2}{2}\right)\frac{70}{57{\cdot}3} - 2 \times 0{\cdot}658 \times 0{\cdot}2 (\sin 90 - \sin 20) - \frac{0{\cdot}2^2}{4}\sin 40\right]$$
$$= \underline{\underline{0{\cdot}0014}}$$

$$I \text{ arc } NO = 0{\cdot}018 \times 0{\cdot}5\left[\frac{0{\cdot}5^2}{2} \times \frac{70}{57{\cdot}3} - \frac{0{\cdot}5^2}{4}\sin 40\right]$$
$$= \underline{\underline{0{\cdot}0010}}$$

$$I \text{ of half web} = 0{\cdot}0060 + 0{\cdot}0030 + 0{\cdot}0024 + 0{\cdot}0021 + 0{\cdot}0014 + 0{\cdot}0010$$
$$= \underline{\underline{0{\cdot}0155}}$$

$$I \text{ of two webs} = 0{\cdot}0155 \times 4$$
$$= \underline{\underline{0{\cdot}0620 \text{ in.}^4}}$$

Total Moment of Inertia of section

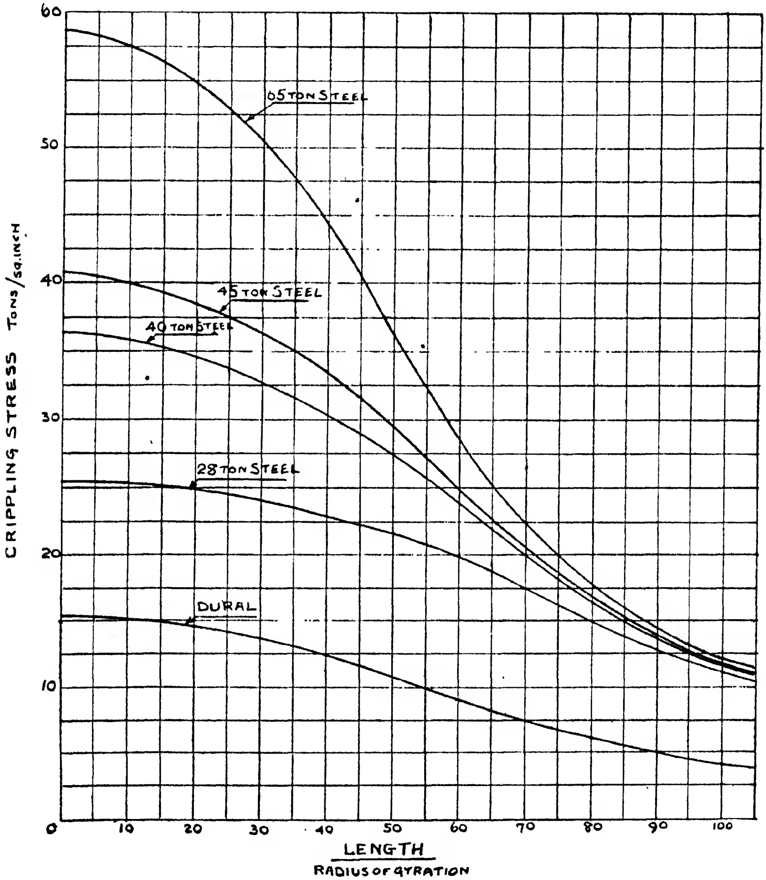
$$= 0{\cdot}2368 + 0{\cdot}0620$$
$$= \underline{\underline{0{\cdot}2988 \text{ in.}^4}}$$

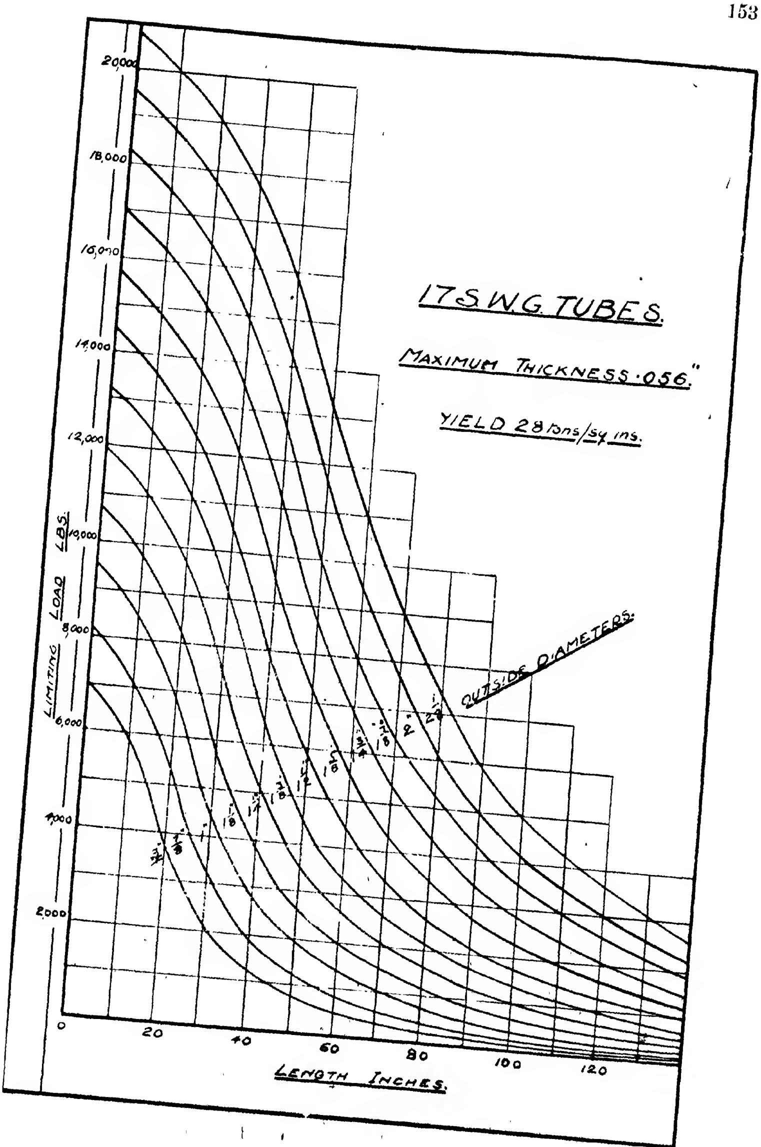
APPENDIX III

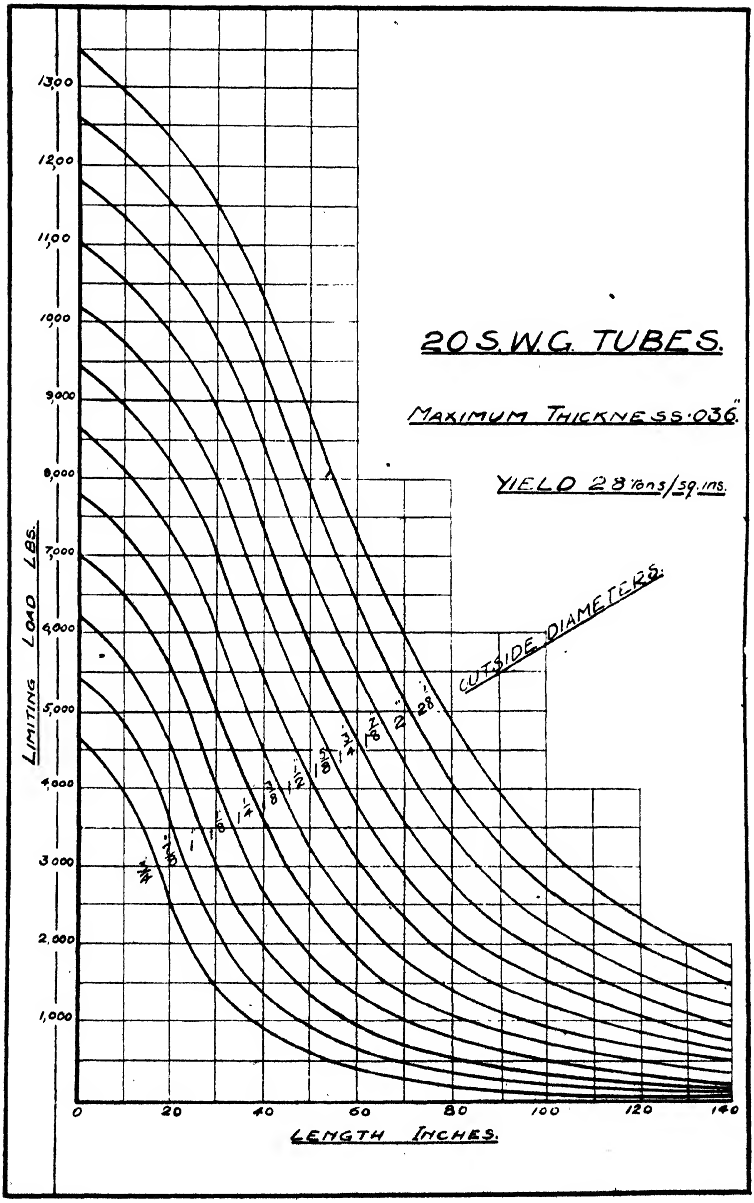
TABLE SHOWING SECTIONAL AREA, MOMENT OF INERTIA ABOUT THE NEUTRAL AXIS, AND RADIUS OF GYRATION OF TUBES

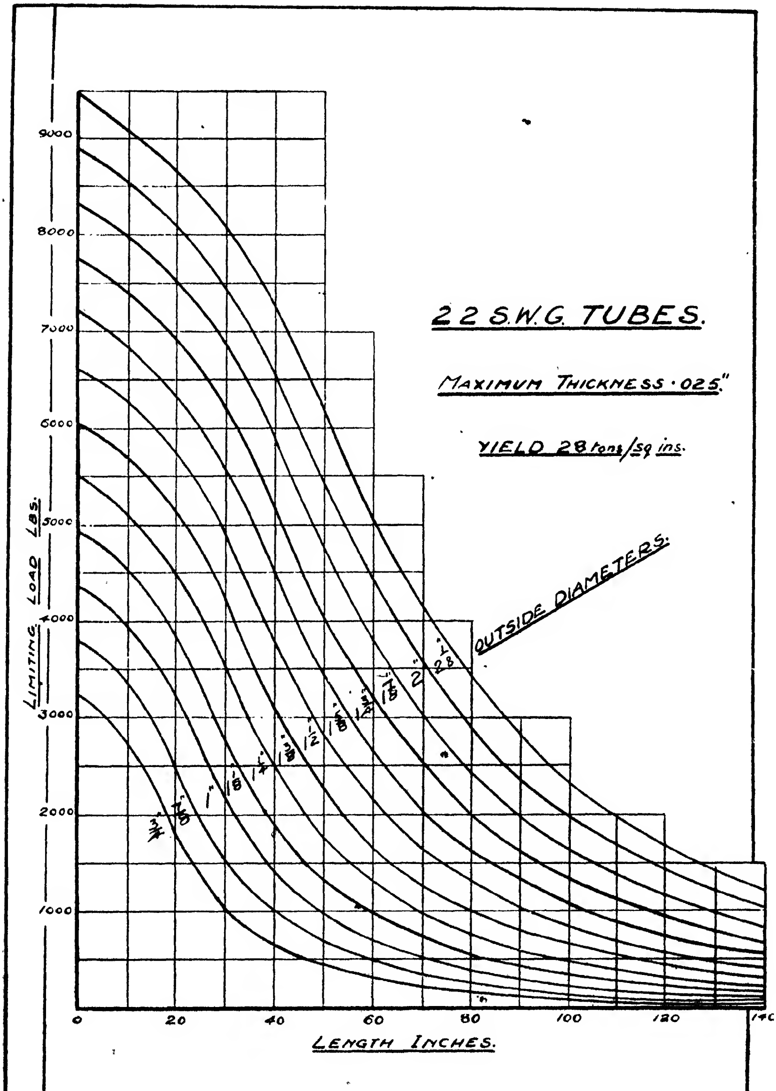
Outside diameter		3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	1 5/8"	1 3/4"	1 7/8"	2"	2 1/8"	2 1/4"
Thickness														
24 S.W.G. (0·022")	A	0·050	0·059	0·068	0·076	0·085	0·094	0·102	0·111	0·119	0·128	0·137	0·146	0·154
	I	0·0033	0·0054	0·0081	0·0116	0·0161	0·0215	0·0279	0·0356	0·0445	0·0548	0·0667	0·0802	0·0955
	K	0·257	0·303	0·346	0·391	0·436	0·478	0·523	0·567	0·611	0·655	0·697	0·740	0·788
22 S.W.G. (0·028")	A	0·0635	0·0745	0·0855	0·0965	0·1075	0·1185	0·1295	0·1405	0·1515	0·1625	0·1735	0·1845	0·1955
	I	0·0042	0·0067	0·0101	0·0145	0·0201	0·0269	0·0351	0·0448	0·0562	0·0693	0·0844	0·1015	0·1207
	K	0·255	0·300	0·344	0·387	0·432	0·475	0·520	0·565	0·608	0·653	0·697	0·742	0·786
20 S.W.G. (0·036")	A	0·081	0·095	0·109	0·123	0·137	0·1514	0·1655	0·180	0·194	0·208	0·222	0·236	0·250
	I	0·005	0·0083	0·0127	0·0183	0·0254	0·0340	0·0442	0·0567	0·0711	0·0879	0·1071	0·1289	0·1533
	K	0·249	0·296	0·341	0·386	0·430	0·473	0·517	0·562	0·606	0·650	0·695	0·739	0·783
18 S.W.G. (0·048")	A	0·106	0·125	0·144	0·162	0·181	0·200	0·219	0·238	0·257	0·2755	0·294	0·313	0·332
	I	0·0065	0·0107	0·0163	0·0236	0·0328	0·0440	0·0577	0·0740	0·0931	0·1150	0·1404	0·1690	0·2014
	K	0·248	0·293	0·336	0·381	0·426	0·470	0·513	0·558	0·602	0·647	0·691	0·735	0·778
17 S.W.G. (0·056")	A	0·122	0·144	0·166	0·188	0·210	0·232	0·254	0·276	0·298	0·320	0·342	0·364	0·386
	I	0·0074	0·0121	0·0185	0·0269	0·0376	0·0506	0·0663	0·0851	0·1070	0·1324	0·1616	0·1949	0·2324
	K	0·246	0·290	0·333	0·378	0·423	0·467	0·510	0·555	0·599	0·644	0·688	0·732	0·774
16 S.W.G. (0·064")	A	0·138	0·163	0·188	0·213	0·238	0·264	0·289	0·314	0·339	0·364	0·389	0·414	0·440
	I	0·0082	0·0135	0·0207	0·0301	0·0421	0·0567	0·0746	0·0958	0·1208	0·1495	0·1826	0·2203	0·2628
	K	0·244	0·288	0·331	0·376	0·420	0·464	0·508	0·553	0·597	0·641	0·685	0·730	0·771
0·025"	A	0·057	0·067	0·077	0·086	0·096	0·106	0·116	0·126	0·135	0·145	0·155	0·165	0·174
	I	0·0037	0·0061	0·0091	0·0131	0·0181	0·0242	0·0315	0·0403	0·0504	0·0622	0·0759	0·0909	0·1070
	K	0·256	0·302	0·345	0·390	0·434	0·477	0·522	0·566	0·610	0·654	0·699	0·741	0·787

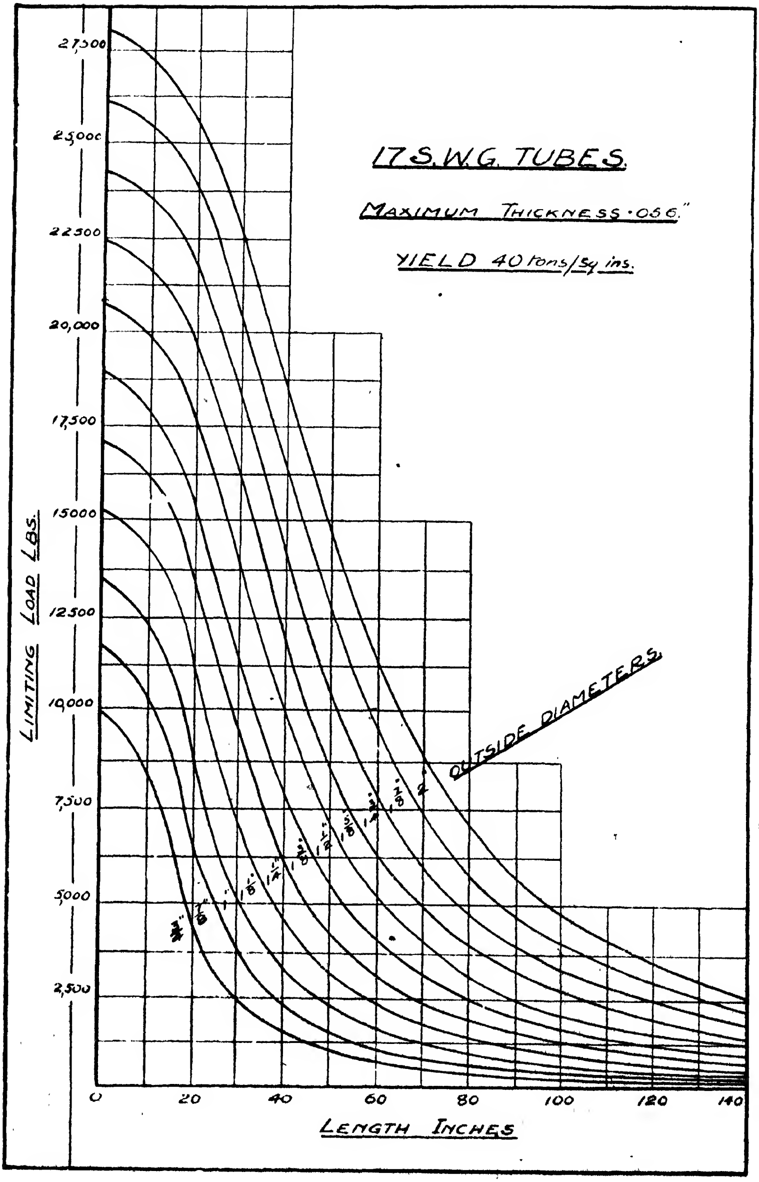
STRUT CURVES FOR 65, 45 40 AND 28 TONS/□" YIELD STEEL AND DURALUMIN



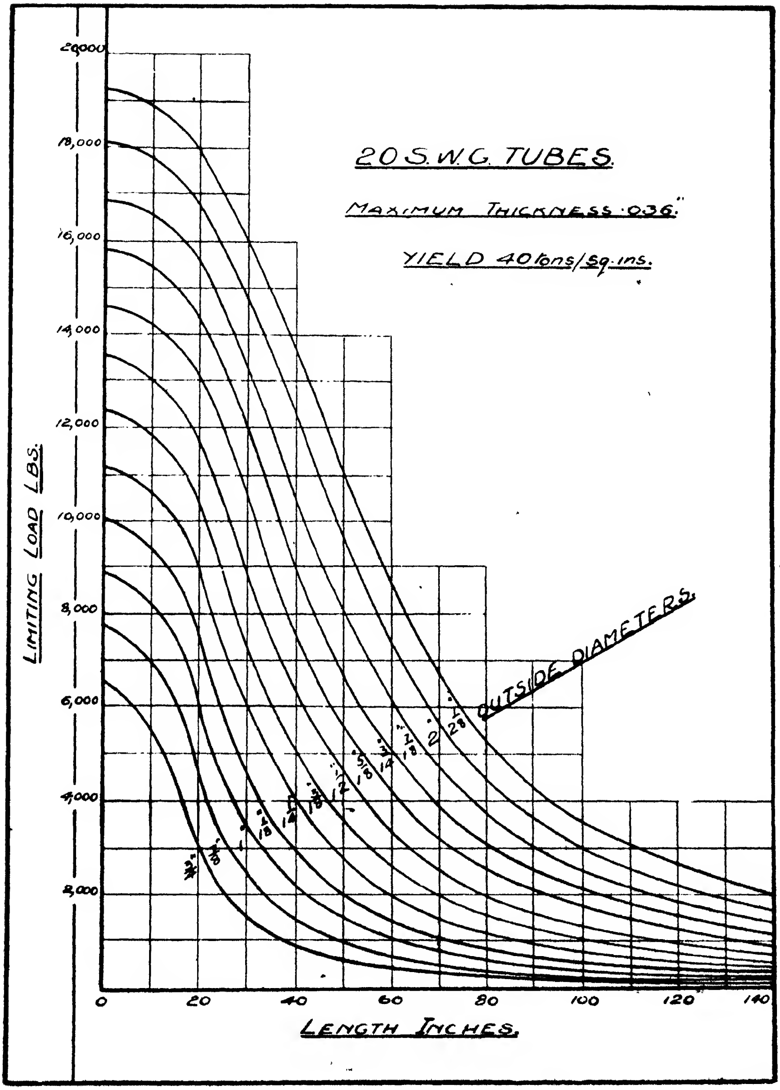

17 S.W.G. TUBES.
MAXIMUM THICKNESS ·056."
YIELD 28 tons/sq ins.
OUTSIDE DIAMETERS.
LIMITING LOAD LBS.
LENGTH INCHES.
20,000
18,000
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
20
40
60
80
100
120


20 S.W.G. TUBES.
MAXIMUM THICKNESS ·036"
YIELD 28 Tons/sq. ins.
OUTSIDE DIAMETERS.
LIMITING LOAD LBS.
LENGTH INCHES.
13,00
12,00
11,00
10,00
9,000
8,000
7,000
6,000
5,000
4,000
3 000
2,000
1,000
0
20
40
60
80
100
120
140
3/4
7/8
1
1 1/8
1 1/4
1 3/8
1 1/2
1 5/8
1 3/4
1 7/8
2"
2 1/8"

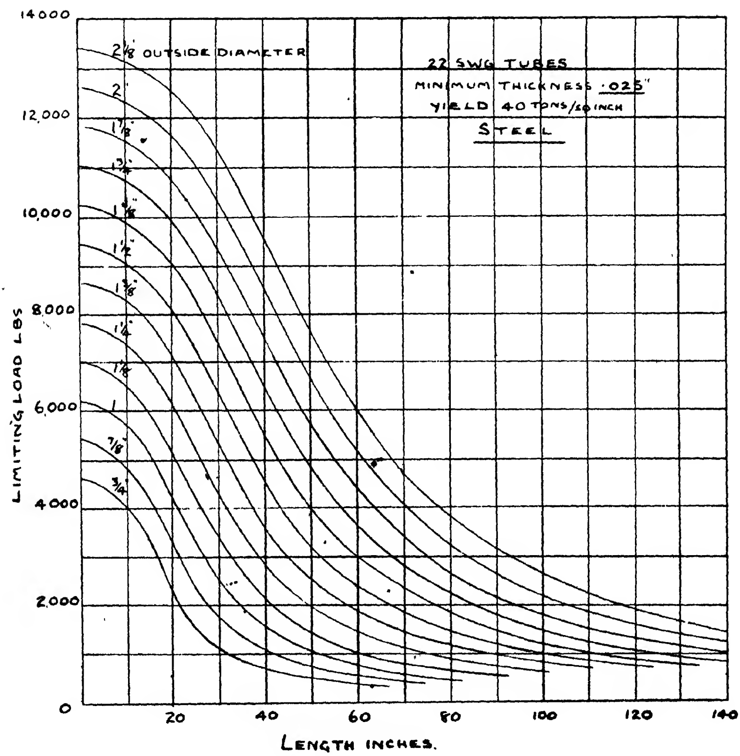

22 S.W.G. TUBES.
MAXIMUM THICKNESS ·025."
YIELD 28 tons/sq ins.
OUTSIDE DIAMETERS.
LIMITING LOAD LBS.
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
20
40
60
80
100
120
140
LENGTH INCHES.
3/4"
7/8"
1"
1 1/8"
1 1/4"
1 3/8"
1 1/2"
1 5/8"
1 3/4"
1 7/8"
2"
2 1/8"

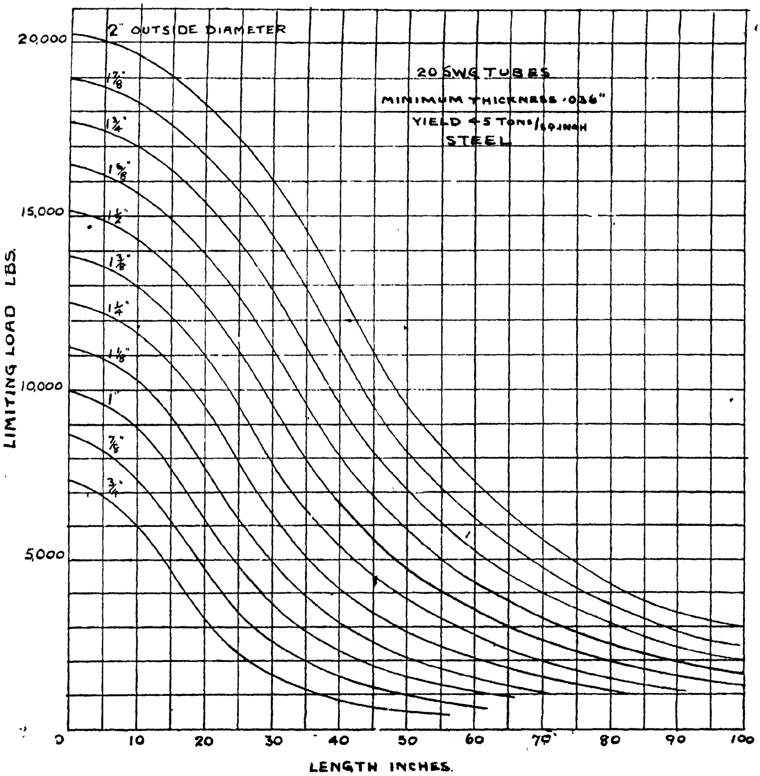


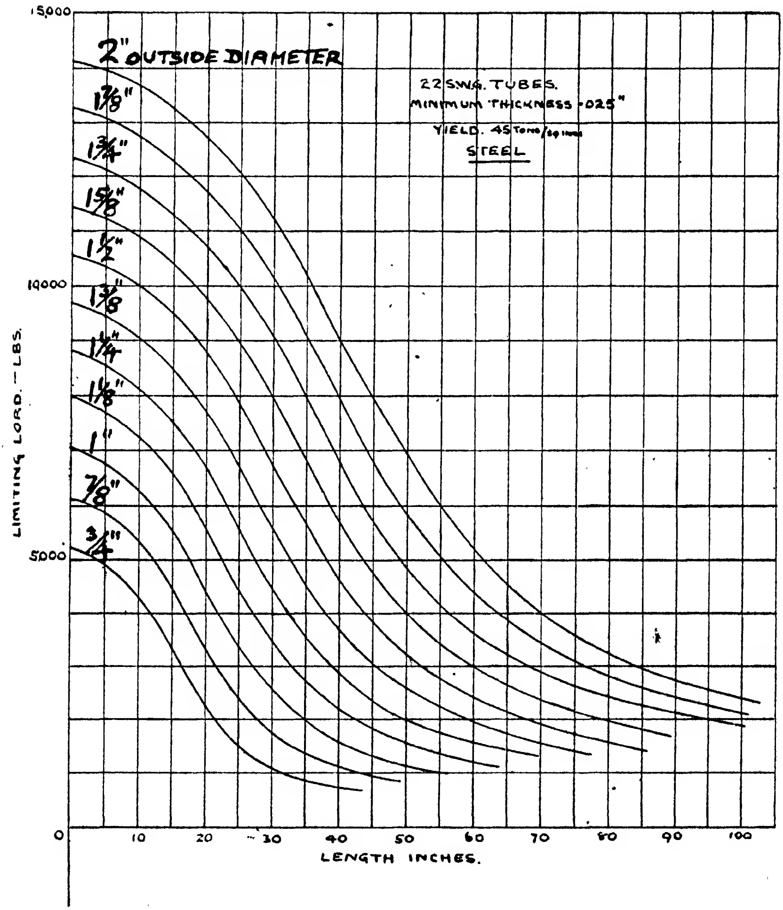
17 S.W.G. TUBES.
MAXIMUM THICKNESS ·056."
YIELD 40 tons/sq ins.
OUTSIDE DIAMETERS.
LIMITING LOAD LBS.
LENGTH INCHES
27,500
25,000
22500
20,000
17,500
15000
12500
10,000
7,500
5000
2,500
0
20
40
60
80
100
120
140

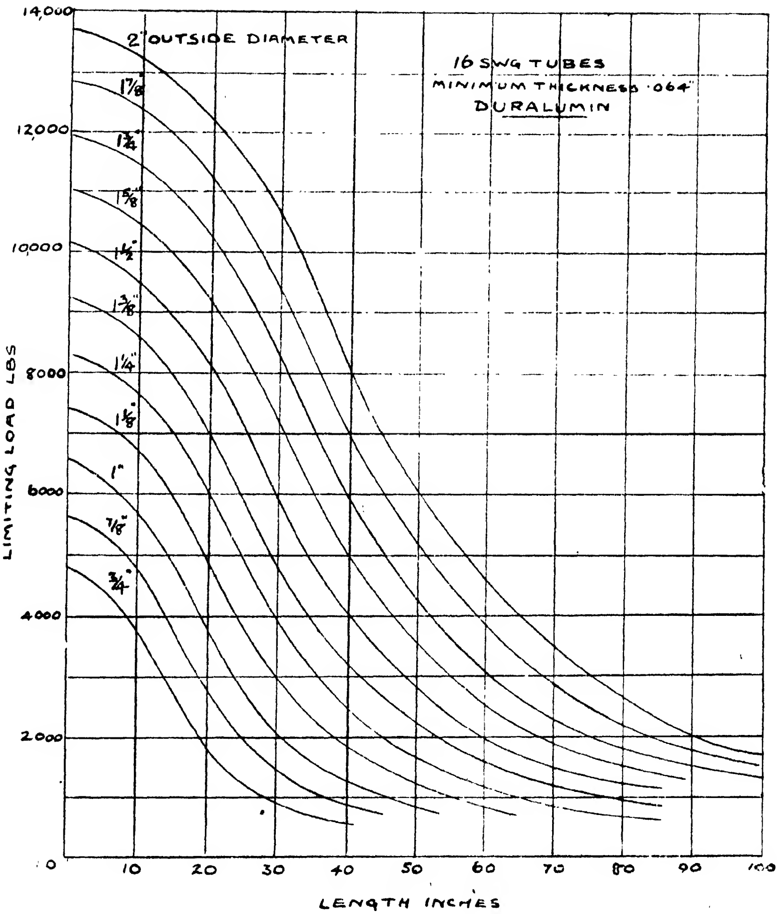


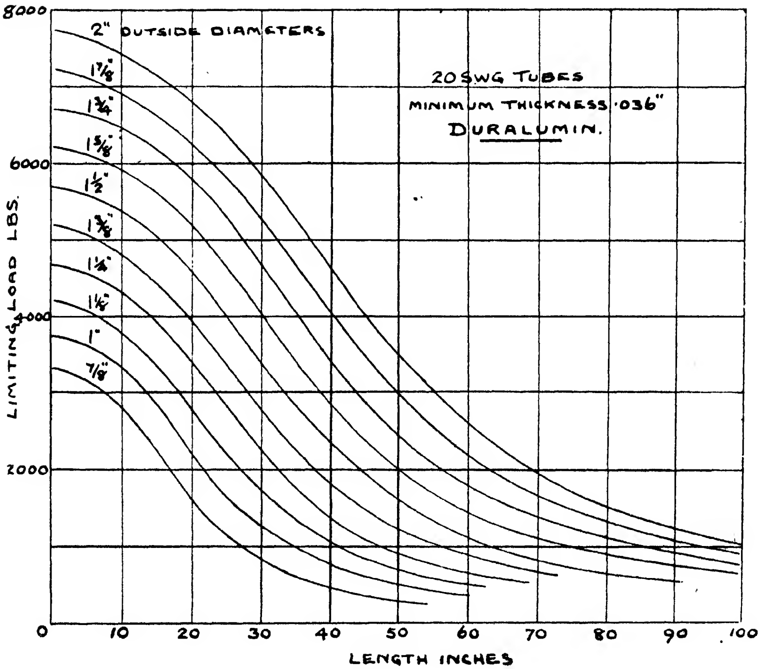
20 S.W.G. TUBES.
MAXIMUM THICKNESS ·036"
YIELD 40 tons/sq. ins.
OUTSIDE DIAMETERS.
LIMITING LOAD LBS.
20,000
18,000
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
20
40
60
80
100
120
140
LENGTH INCHES.


2⅛" OUTSIDE DIAMETER
2"
1⅞"
1¾"
1⅝"
1½"
1⅜"
1¼"
1⅛"
1"
⅞"
¾"
22 SWG TUBES
MINIMUM THICKNESS ·025"
YIELD 40 TONS/SQ INCH
STEEL
LIMITING LOAD LBS
14000
12,000
10,000
8,000
6,000
4000
2,000
0
20
40
60
80
100
120
140
LENGTH INCHES.


2" OUTSIDE DIAMETER
1 7/8"
1 3/4"
1 5/8"
1 1/2"
1 3/8"
1 1/4"
1 1/8"
1"
7/8"
3/4"
20 SWG TUBES
MINIMUM THICKNESS ·036"
YIELD 45 TONS/SQ.INCH
STEEL
20,000
15,000
10,000
5,000
LIMITING LOAD LBS.
0
10
20
30
40
50
60
70
80
90
100
LENGTH INCHES.


2" OUTSIDE DIAMETER
1 7/8"
1 3/4"
1 5/8"
1 1/2"
1 3/8"
1 1/4"
1 1/8"
1"
7/8"
3/4"
22 S.W.G. TUBES.
MINIMUM THICKNESS ·025"
STEEL
15,000
10,000
5000
0
10
20
30
40
50
60
70
80
90
100
LENGTH INCHES.
LIMITING LOAD. – LBS.


2" OUTSIDE DIAMETER
1⅞"
1¾"
1⅝"
1½"
1⅜"
1¼"
1⅛"
1"
⅞"
¾"
16 SWG TUBES
MINIMUM THICKNESS ·064"
DURALUMIN
14,000
12,000
10,000
8000
6000
4000
2000
0
10
20
30
40
50
60
70
80
90
100
LIMITING LOAD LBS
LENGTH INCHES


2" OUTSIDE DIAMETERS
1⅞"
1¾"
1⅝"
1½"
1⅜"
1¼"
1⅛"
1"
⅞"
20 SWG TUBES
MINIMUM THICKNESS ·036"
DURALUMIN.
8000
6000
4000
2000
0
LIMITING LOAD LBS.
10
20
30
40
50
60
70
80
90
100
LENGTH INCHES

APPENDIX IV

PRINCIPLE OF LEAST WORK

FORCES in redundant structures cannot be found by the ordinary methods referred to in this book. It is the object of this appendix to introduce a method of solution normally requiring higher mathematics than has been assumed hitherto, but which is essential for the complete stressing of aircraft structures.

When a member is stressed within the elastic limit, work is done on it, and it stores energy, called strain energy, which is given out when the stress is removed. Work done on a tensile member is the product of the average force and extension. Within the limit of proportionality this work equals—

$$\frac{\text{Pull} \times \text{extension}}{2}$$

(i.e. the area under the load/extension graph).

The Principle of Least Work states that if the structure is capable of taking loads in more than one way, the stresses in the members will be such that the work done is a minimum. This requires the cross-sectional area and material of the members to be known or estimated. If an estimated size is found to be wrong, a second attempt will be necessary.

We will first apply the principle to a structure that can be solved by usual methods.

EXAMPLE. A symmetrical block weighing 3000 lb. is supported by three wires of equal diameter and length. One wire, of steel, is in the centre, and there is a brass wire at each end. If Young's Modulus for steel is 13,000 tons/sq. in., and for brass 6000 tons/sq. in., find the load taken by each wire. Any deflection of the block to be neglected.

By usual method

Let P_B = Pull in brass wires

P_s = Pull in steel wire

then $2P_B + P_s = 3000$

$P_s = 3000 - 2P_B$

Elongation of each wire is the same

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{PL}{Ae}$$

therefore, elongation $(e) = \dfrac{PL}{AE}$

Elongation of brass wire = elongation of steel wire

$$\frac{P_B L}{AE_B} = \frac{P_S L}{AE_S}$$

$$\frac{P_B L}{AE_B} = \frac{(3000 - 2P_B) L}{AE_S}$$

$$\frac{3000 - 2P_B}{P_B} = \frac{E_S}{E_B}$$

$$\frac{3000}{P_B} = \frac{E_S}{E_B} + 2$$

$$P_B = \frac{3000}{\left(\frac{13000}{6000} + 2\right)} = \frac{3000}{4 \cdot 166}$$

$$= \underline{\underline{720 \text{ lb.}}} \text{ load in brass wire.}$$

$$\text{Load in steel wire} = 3000 - 2 \times 720$$
$$= \underline{\underline{1560 \text{ lb.}}}$$

By Principle of Least Work

$$\text{Work done on each wire} = \frac{\text{Pull} \times \text{extension}}{2}$$

$$\text{Work done on brass wires} = \frac{2P_B \times \frac{P_B L}{AE_B}}{2}$$

$$\text{Work done on steel wire} = \frac{P_S \times \frac{P_S L}{AE_S}}{2}$$

Total work done =

$$W = \frac{P_B^2 L}{AE_B} + \frac{P_S^2 L}{2AE_S}$$

$$= \frac{L}{A}\left[\frac{P_B^2}{6000} + \frac{(3000 - 2P_B)^2}{26000}\right]$$

$$= \frac{L}{A}\left[\frac{P_B^2}{6000} + \frac{9000000 - 12000\, P_B + 4P_B^2}{26000}\right]$$

$$= \frac{L}{A}\left[\frac{8 \cdot 333 P_B^2 + 9000000 - 12000 P_B}{26000}\right]$$

Readers who understand calculus will know that W will be a minimum when—

$$\frac{dW}{dP} = 0$$

$$\frac{dW}{dP} = \frac{L}{A26000}\left[16{\cdot}667P_B - 12000\right] = 0$$

therefore $P_B = \dfrac{12000}{16{\cdot}667}$

$= 720$ lb.

and as before $P_s = \underline{\underline{1560 \text{ lb.}}}$

Those who do not understand calculus may obtain the answer by plotting a graph of W against P_B, and find from it the value of P_B when W is a minimum.

As the constants will not affect the resulting value of P_B, they may be left out, and values of

$(8{\cdot}333P_B^2 - 12{,}000P_B)$ plotted against P_B as follows—

P_B	200	500	800	1000	700
$8{\cdot}333P_B^2$	333,000	2,080,000	5,330,000	8,330,000	4,080,000
$-12000P_B$	2,400,000	6,000,000	9,600,000	12,000,000	8,400,000
Total	− 2,067,000	− 3,920,000	− 4,270,000	− 3,670,000	− 4,320,000

The graph (Fig. 106) shows that work done is a minimum when P_B equals 720 lb.

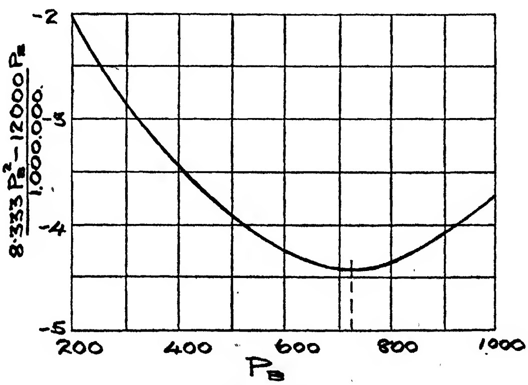


FIG. 106

We will apply this principle to a simple rectangular frame, Fig. 107. It is assumed that all members will resist both tension and compression.

AB is 2 in. $\times$ 2 in. square
BC ,, 1 in. $\times$ 1 in. square
CD ,, 2 in. $\times$ 2 in. square
AC ,, $2\frac{1}{2}$ in. $\times$ $2\frac{1}{2}$ in. square
BD ,, $2\frac{1}{2}$ in. $\times$ $2\frac{1}{2}$ in. square

The material is spruce, $E = 1600000$ lb./sq. in.

Length of BD and $AC = \sqrt{6^2 + 4^2} = \sqrt{52}$.

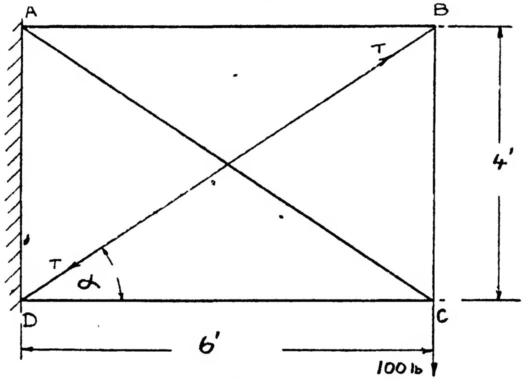


FIG. 107

Replace the redundant member BD by the forces T at B and D, T being the force in this member.

From the polygon or triangle of forces for each joint we have—

$$\text{Load in } BC = T \sin \alpha = \frac{T4}{\sqrt{52}} \text{ Tension}$$

$$\text{Load in } AB = T \cos \alpha = \frac{T6}{\sqrt{52}} \text{ Tension}$$

$$\text{Load in } AC = \frac{100 - BC}{\sin \alpha} = \frac{100 - \dfrac{T4}{\sqrt{52}}}{\dfrac{4}{\sqrt{52}}}$$

$$= 25\sqrt{52} - T. \text{ Tension}$$

$$\text{Load in } CD = AC \cos \alpha = \left(25\sqrt{52} - T\right)\frac{6}{\sqrt{52}}$$

$$= 150 - \frac{T6}{\sqrt{52}} \text{ Compression}$$

Increase or decrease in length $= \dfrac{PL}{AE}$

Work done in straining = average load × increase or decrease in length.

Average load $= \dfrac{P}{2}$

Work done $= \dfrac{P^2L}{2AE}$, where P equals load in member.

Now find the work done on each member in terms of T, and from this the total work done.

Member	P Load lb.	A Area sq. in.	L Length ft.	Work done ft. lb. $= \dfrac{P^2L}{2AE}$
BD	T	$6\frac{1}{4}$	$\sqrt{52}$	$\dfrac{1}{2E}\left[\dfrac{T^2 \times \sqrt{52} \times 16}{100}\right]$
BC	$\dfrac{T4}{\sqrt{52}}$	1	4	$\dfrac{1}{2E}\left[\dfrac{16T^2 \times 4}{52 \times 1}\right]$
AB	$\dfrac{T6}{\sqrt{52}}$	4	6	$\dfrac{1}{2E}\left[\dfrac{36T^2 \times 6}{52 \times 4}\right]$
AC	$25\sqrt{52} - T$	$6\frac{1}{4}$	$\sqrt{52}$	$\dfrac{1}{2E}\left[\dfrac{\sqrt{52} \times 16}{100}(25^2 \times 52 - 50\sqrt{52}T + T^2)\right]$
CD	$150 - \dfrac{T6}{\sqrt{52}}$	4	6	$\dfrac{1}{2E}\left[\dfrac{6}{4}\left(150^2 - \dfrac{1800}{\sqrt{52}}T + \dfrac{36T^2}{52}\right)\right]$

Total work (W) = sum of last column.

$$= \frac{1}{2E}\left[\quad\left(\frac{16\sqrt{52}}{100} + \frac{64}{52} + \frac{54}{52} + \frac{16\sqrt{52}}{100} + \frac{54}{52}\right)\right.$$

$$\left. - T\left(416 + \frac{2700}{\sqrt{52}}\right) + \frac{16\sqrt{52}}{100} \times 25^2 \times 52 + \frac{6}{4} \times 150^2\right]$$

$$= \frac{1}{2E}\left[T^2\left(0{\cdot}32\sqrt{52} + \frac{172}{52}\right) - T\,(416 + 374) + 43500 + 33750\right]$$

$$= \frac{1}{2E}\left[5{\cdot}62T^2 - 790T + 77250\right]$$

The principle states that T must be such a value that W is a minimum. Readers who are unable to do calculus may plot a graph of W against T, and find the value of T from the lowest part of the graph, as in the previous example. Those who wish to use this method of solving

structures must learn calculus, and replace the graph by the following simple calculation.

$$W \text{ is a minimum when } \frac{dW}{dT} = 0$$

$$W = \frac{1}{2E}\left[5{\cdot}62T^2 - 790T + 77250\right]$$

$$\frac{dW}{dT} = \frac{2T}{2E} \times 5{\cdot}62 - \frac{790}{2E} = 0$$

$$2 \times 5{\cdot}62T = 790$$

$$T = \frac{790}{2 \times 5{\cdot}62} = 70{\cdot}3 \text{ lb.}$$

The loads in all the members may now be found, as we know them in terms of T.

Load in $BD = 70{\cdot}3$ lb.

,, $BC = \dfrac{70{\cdot}3 \times 4}{\sqrt{52}} = 39$ lb.

,, $AB = \dfrac{70{\cdot}3 \times 6}{\sqrt{52}} = 58\frac{1}{2}$ lb.

,, $AC = 25\sqrt{52} - 70{\cdot}3 = 110$ lb.

,, $CD = 150 - \dfrac{70{\cdot}3 \times 6}{\sqrt{52}} = 91\frac{1}{2}$ lb.

The loads in any redundant frame may be found in this way, by replacing the redundant members by forces, and finding the loads in the other members in terms of those forces. When the members are an imperfect fit, the problem becomes too complicated for inclusion here.

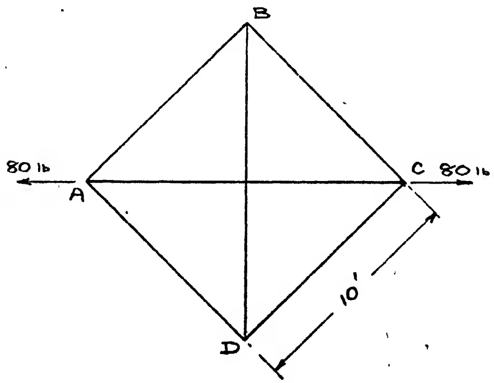


FIG. 108

EXAMPLE 1. Two bars, of the same material and length, run parallel to the same joints in a structure. The cross-sectional area of one is 0·5 sq. in. and of the other 0·7 sq. in. If the total load carried is 24,000 lb. find, by the *Principle of Least Work*, the load in each.

Ans. 0·5 sq. in. bar 10,000 lb.
0·7 ,, ,, 14,000 lb.

EXAMPLE 2. Find the forces in the members of the redundant square frame shown in Fig. 108, when subjected to the two external loads of

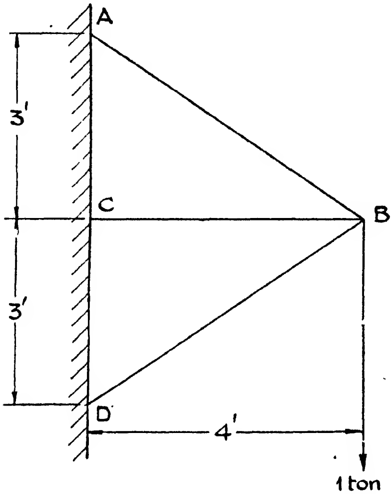


FIG. 109

80 lb. Given: all members are 2 in. × $1\frac{1}{2}$ in. E for BD and AC = 1·5 × 10^6 lb./sq. in. and E for the side members = 2·0 × 10^6 lb./sq. in.

Ans. AC = 54 lb. tension
BD = 26 lb. compression
Sides = 18·5 lb. tension

EXAMPLE 3. Find the load in the member CB of the redundant frame in Fig. 109, if all members are of the same material but the cross-sectional area of AB is twice the cross-sectional areas of BD and CB.

Ans. 0·19 ton.

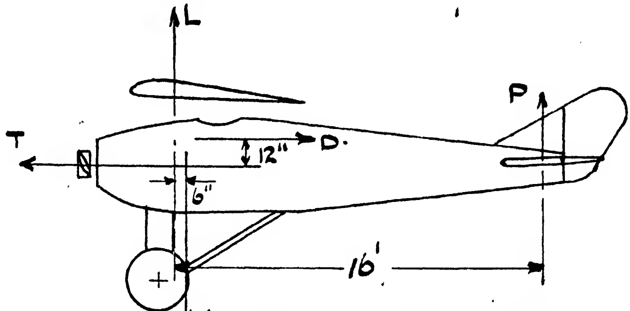
EXAMPLES

1. A plank spanning a chasm is freely supported at its ends, which are 12 ft. apart. It has three men on it, each weighing 150 lb. The plank weighs 10 lb./ft. One man is 2 ft. from one side and the other two are respectively 6 ft. and 8 ft. from the same side.

Find the reactions of the supports.

Ans. 310 lb. and 260 lb.

2. Find the drag D, lift L, and tail load P for the aeroplane shown in the sketch, if the weight W = 3000 lb., the thrust T = 260 lb., and the machine is in steady flight.



Sketch for Problem No. 2.

Ans. D = 260 lb., L = 2890 lb., P = 110 lb.

3. A captive balloon is acted upon by an horizontal force of 300 lb., due to the wind pressure. The buoyancy force on the balloon is 1000 lb. and its weight is 400 lb. Find the tension in the holding-down cable and its angle to the horizontal. (Neglect the weight of the cable.)

Ans. 671 lb., 63° 26′.

4. The forces acting on a certain aeroplane in steady horizontal flight are—

(i) Its weight, 8000 lb.
(ii) The airscrew thrust, 900 lb. acting horizontally through the C.G.
(iii) The lift, acting through the C.P. which is 6 in. behind and 12 in. above the C.G.
(iv) The active drag.
(v) The passive drag 0·8 active drag, acting 9 in. below the C.G.
(vi) A vertical tail load, acting 25 ft. behind the C.G.

Calculate the lift, the tail load, and the passive drag.

Ans. L = 8155 lb., P = 155 lb., D = 400 lb.

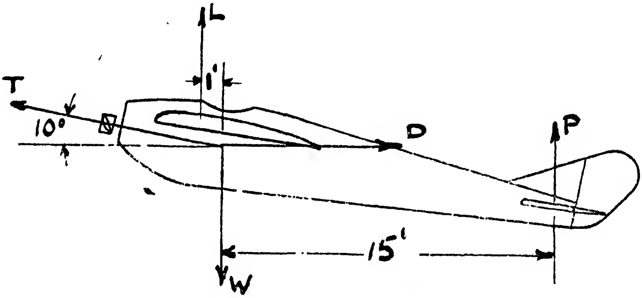
5. A weight of 50 lb. is suspended from two points A and B on a horizontal ceiling. A being 10 ft. from B. The suspension cord from A

is 12 ft. long and that from *B* 14 ft. long. Find the pull (or tension) in the cords.
Ans. 14 lb. 38½ lb.

6. Five forces acting at a point are in equilibrium.
(1) is 5 lb. (2) 6 lb. at 20° to (1), (3) 2 lb. at 70° to (1), and (4) 6 lb. at 270° to (1).
Find the magnitude and direction of the remaining force.
Ans. 11½ lb. at 170° to (1)

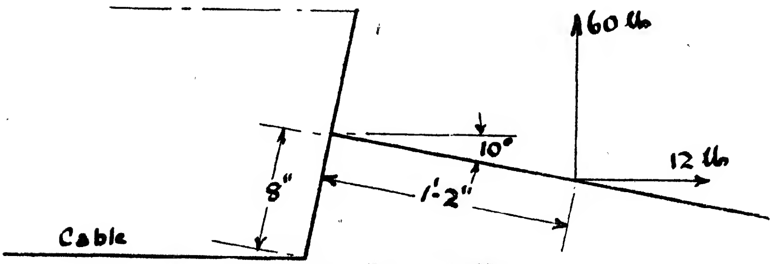
7. A force of 12 lb. acts at 45° to the vertical. Resolve it into two components, one vertical and one at 120° to the vertical.
Ans. 13·3 lb. 9·8 lb.

8. The aeroplane in the figure is maintained in steady horizontal flight by the forces *T*, *W*, *L*, *D*, and *P*. Given T = 360 lb., and W = 4000 lb., find *L*, *D*, and *P*.



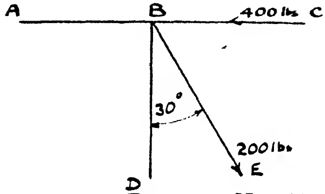
SKETCH FOR PROBLEM No. 8
Ans. L = 3691 lb., D = 355 lb., P = 246 lb.

9. On a certain aeroplane the rudder control cable is attached to the rudder lever at a distance of 8 in. from the rudder hinge. The centre of pressure of the rudder is 1 ft. 2 in. from the hinge, and the components of the resultant air pressure when the rudder is set at 10° are shown in the sketch. Find the pull in the cable.



SKETCH FOR PROBLEM No. 9.
Ans. 109 lb.

10. Find the loads in the members AB and BD for the fuselage joint shown in the sketch.



Sketch for Problem No. 10

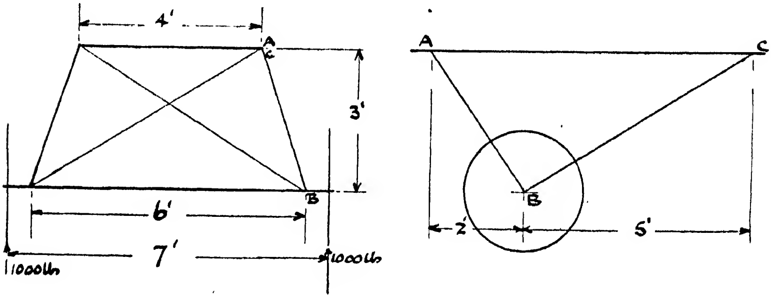
Ans. 300 lb., 173 lb. compression.

11. Explain the difference between deficient, perfect, and redundant frameworks, showing by sketches two examples of each.

12. For the chassis shown in figure, find—

(a) The maximum bending moment in the axle.

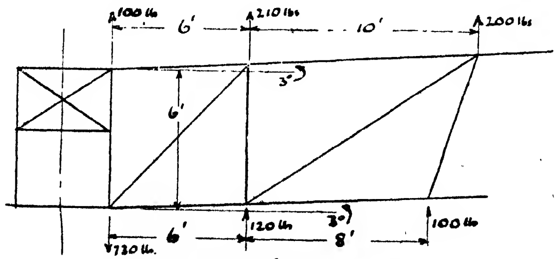
(b) The loads in the members AB and BC.



Sketch for Problem No. 12

Ans. (a) 500 ft.-lb.; (b) 891 lb., 563 lb.

13. (a) Find the loads in the wing structure of the biplane shown in the sketch.

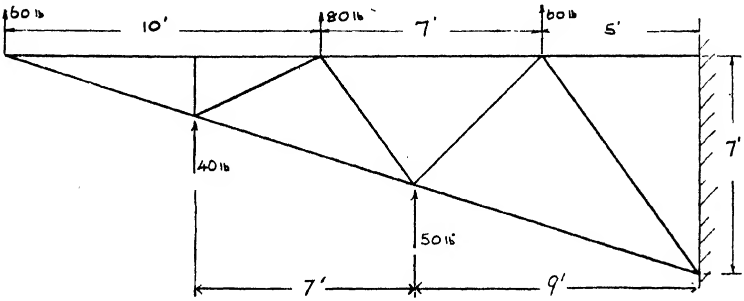


Sketch for Problem No. 13

(*b*) Check the load in the top spar, outer bay, by means of the method of sections.

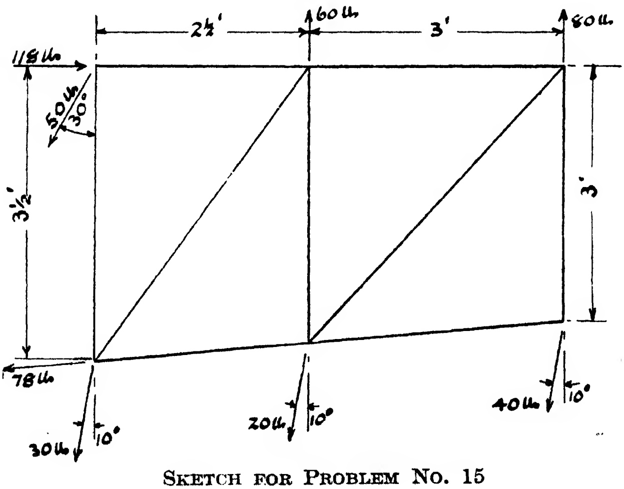
Ans. (*b*) 467 lb.

14. Find the forces in the members of the structure shown in the sketch.



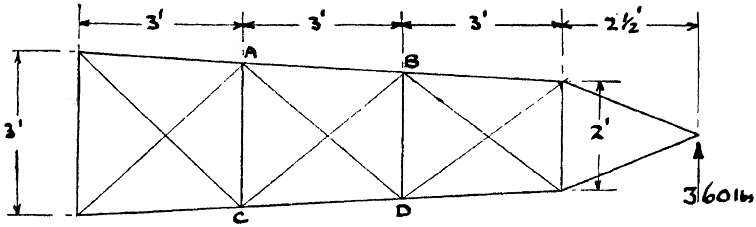
SKETCH FOR PROBLEM No. 14

15. Draw a stress diagram of the fuselage end shown in the sketch, and from it find the magnitude and nature of the loads in the members.



SKETCH FOR PROBLEM No. 15

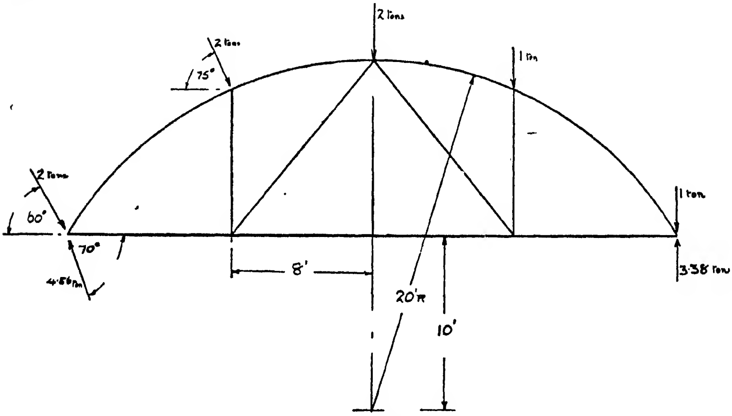
16. The sketch shows the rear horizontal bracing of a fuselage loaded in a turn. Find by the method of section the loads in the members *AB*, *BC*, *BD*, and *CD*.



SKETCH FOR PROBLEM No. 16

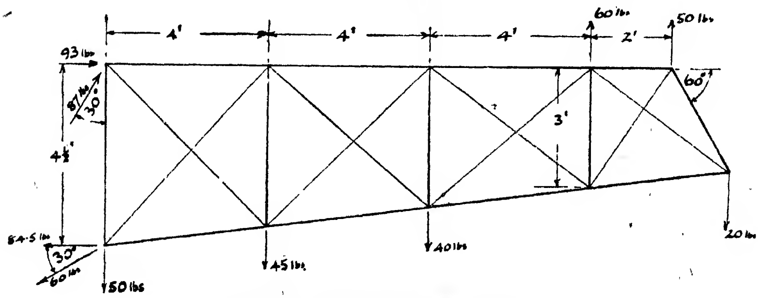
Ans. *AB*, 1152 lb. compression.
BC, 390 lb. tension.
BD, 264 lb. compression.
CD, 852 lb. tension.

17. Find the forces in the members of the curved roof truss in the sketch.



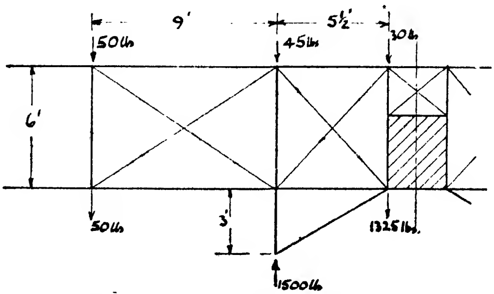
SKETCH FOR PROBLEM No. 17

18. Find the loads in the members of the fuselage structure shown in the sketch. (*Note.* The cross-bracing will only take tension.)



SKETCH FOR PROBLEM No. 18

19. The external landing loads on a wing truss are given in the sketch. Find the loads in the spars and wires.



SKETCH FOR PROBLEM No. 19

20. Discuss the factors which have contributed to the advance in aircraft performance in recent years.

21. What do you understand by the term Load Factor? An aeroplane is in steady horizontal flight at a speed of 100 m.p.h.; by a rapid movement of the controls the angle of incidence is increased without any appreciable loss of speed. If the speed in steady horizontal flight at this angle is 50 m.p.h., find the proof factor if this is a maximum manoeuvre appropriate to the type.

Ans. 6.

22. An aeroplane is found to have its C.G. farther back than was estimated.

(*a*) What effect will this have on the strength and flying of the aeroplane?

(*b*) State two practical methods by which this may be corrected.

23. An aeroplane when weighed in rigging position is found to have a reaction on the wheels of 2000 lb., and on the tail skid of 160 lb. If the horizontal distance between the wheels and skid is 20 ft., find the distance of the C.G. behind the wheels.

How far would the C.G. move forward if a load of 180 lb. was moved from 4 ft. behind to 8 ft. in front of the wheels?

Ans. 17·8 in.; 1 ft.

24. In an experiment for finding the C.G. of an aeroplane the following results were obtained—

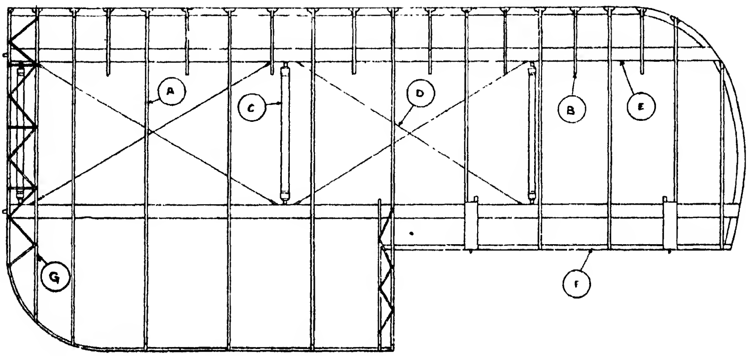
Reaction on wheels in rigging position	4120 lb.
Reaction on wheels in tail down position	4000 lb.
Total weight of machine	4200 lb.
Horizontal distance wheels to skid in rigging position	21 ft.
Horizontal distance wheels to skid with tail down	21·12 ft.

Height of skid above ground in rigging position 4 ft.
Diameter of landing wheels . . . . 3 ft.

Find the horizontal distance behind the wheels and height above the ground of the C.G. of the machine in rigging position.

Ans. 0·4 ft.; 4·8 ft.

25. Name and state the function of each of the components marked *A*, *B*, *C*, *D*, *E*, *F*, and *G* of the wing structure in the sketch.



SKETCH FOR PROBLEM No. 25

26. What functions do the following parts of an aeroplane fulfil?

(*a*) Compression ribs.
(*b*) Inter-plane struts.
(*c*) Undercarriage radius rod.
(*d*) Incidence wires.

27. Explain how the air reactions on the wings are transferred from the fabric to the fuselage.

28. Sketch a monocoque fuselage, and name and state the function of the different members.

29. Explain with the aid of sketches the working of any undercarriage shock-absorbing device. What are the main qualities required in such a shock absorber?

30. A flying wire has to carry a load of 800 lb., and an ultimate factor of 6 has to be used. If the ultimate stress for R.A.F. wires is 65 tons/sq. in., calculate the cross-sectional area of the wire.

Ans. 0·033 sq. in.

31. Two flat steel bars 4 in. wide have to be lapped and bolted together in order to form a tension member, to carry a load of 180,000 lb. Find the necessary diameter of the bolt and the thickness of each bar. Take the tensile and shear stress of the bar and bolt to be 32 tons/

sq. in. and 40 tons/sq. in. respectively. What will be the bearing stress?

Ans. $1\frac{5}{8}$ in., 1·05 in., 47·2 tons/sq. in.

32. Two flat steel bars are to be fastened together by a bolt to form a tension member. The bars are 3 in. wide and $\frac{1}{2}$ in. thick, and the tensile stress in the bar is to equal the shear stress in the bolt. Find the diameter of the bolt.

Ans. 1·1 in.

33. The loads in a bracing wire of circular cross-section under three conditions of normal flight—viz. centre of pressure forward, centre of pressure back, and nose dive—are found to be 1800 lb., 1000 lb., and 3000 lb. respectively. If the ultimate factors are 6 for C.P. forward, 7 for C.P. back, and 1·5 for nose dive, determine the necessary diameter of the wire. Given ultimate stress 50 tons/sq. in. Proof stress 45 tons/sq. in.

Ans. 0·35 in.

34. The maximum horse-power which a certain aero engine can develop is 450 at a speed of 1700 r.p.m. The power is transmitted to the propeller by 6 bolts $\frac{1}{2}$ in. diameter passing through the boss and two flanges in the propeller shaft so that the bolts are in double shear. The radius of the bolt circle is 3 in.

Find the torque transmitted to the propeller and the shear stress in the bolts.

Ans. 1390 ft.-lb.; 2367 lb./sq. in.

35. An aeroplane spar has an $\frac{1}{8}$ in. diameter rivet hole punched in it. I the material is 0·04 in. thick and has an ultimate shear strength of 50 tons/sq. in., find—

(*a*) The load in the punch.
(*b*) The stress in the punch.

Ans. 1757 lb.; 64 tons/sq. in.

36. A duralumin tube 3 ft. long is used as a strut and is subjected to a compressive end load of 4500 lb. If the tube is 20 S.W.G. thick, find a suitable diameter.

Ans. $1\frac{7}{8}$ in.

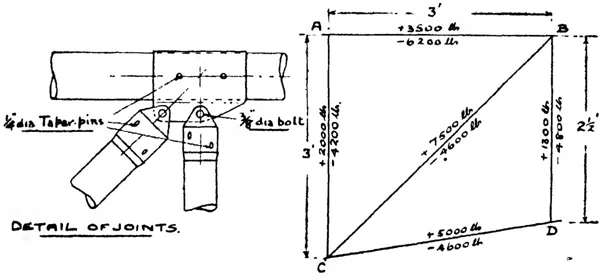
37. A steel tube which is 1 in. diameter and 0·025 in. thick is fitted at each end with a forked socket, by means of two $\frac{3}{16}$ in. diameter taper pins. Find the maximum tensile and bearing stresses in the tube when it is subjected to a tension of 3000 lb.

Ans. 43,000 lb./sq. in.; 160,000 lb./sq. in.

38. A corrugated rolled steel strut is made of 65 tons/sq. in. yield steel. Its radius of gyration is 0·47 in., area of cross-section 0·12 sq. in., and length 30 in. What load will it withstand?

Ans. 6880 lb.

39. The maximum compression and tension loads (including load factors) which will come on the fuselage bay in the sketch are shown against their respective members. The struts which are 20 S.W.G., tubes, may be subjected to a stress of 28 tons/sq. in. Determine suitable diameters.



SKETCH FOR PROBLEM NO. 39

Ans. $AB = 1\frac{1}{4}$ in., $AC = 1$ in., $BD = \frac{7}{8}$ in.,
$BC = 2$ in., $CD = 1\frac{3}{8}$ in.

40. If the size of the members BC and BD, Question 16, are determined by the condition considered, and if the ultimate factor for this condition is to be 4, determine suitable sizes for these members. Ultimate stress in the tie is 60 tons/sq. in. and in the strut 28 tons/sq. in.

Ans. $BC = \frac{1}{8}$ in. diameter.
$BD = \frac{3}{4}$ in. diameter × 22 S.W.G. tube.

41. What is the effect on the strength/weight ratio of—
(*a*) Increasing the length of a strut?
(*b*) Increasing the diameter of a thick strut?
(*c*) Increasing the diameter of a very thin strut?
(*d*) Corrugating a strut with a too large diameter-thickness ratio?

42. The following particulars apply to an aeroplane metal spar which is symmetrical about the neutral axis—
Moment of Inertia of section = 0·3 in.^{4}.
Total area of section = 0·25 sq. in.
Distance of centroid of half section above neutral axis = 1·1 in.
Thickness of web at neutral axis = 0·03 in.
Maximum allowable shear stress = 60 tons/sq. in.
Find the maximum shear force the spar will withstand.

Ans. 3·9 tons.

43. A 10 ft. beam is freely supported at each end and carries a load of 500 lb. evenly distributed over its whole length.
(*a*) Draw the bending moment and shear force diagrams.
(*b*) Find the minimum width, if it is of rectangular cross-section and 2 in. deep. The stress is not to exceed 4 tons/sq. in.

Ans. (*b*) 1·26 in.

44. A beam is tubular in cross-section, 3 in. in diameter and 0·056 in. thick. It carries a load of 1200 lb. in the centre of the span of 8 ft. Find the maximum bending moment, and the maximum stress in the beam. Neglect the weight of the beam.

Ans. 2400 ft.-lb.; 76,900 lb./sq. in.

45. If the beam in Question 44 is also subjected to a compressive end load of 2500 lb., and the total deflection in the centre is 1·4 in., find the maximum stress under this condition.

Ans. 90,900 lb./sq. in.

46. Why must members in a structure be particularly strong when they are to be subjected to bending as well as compressive loads? What parts of an aeroplane carry both types of load?

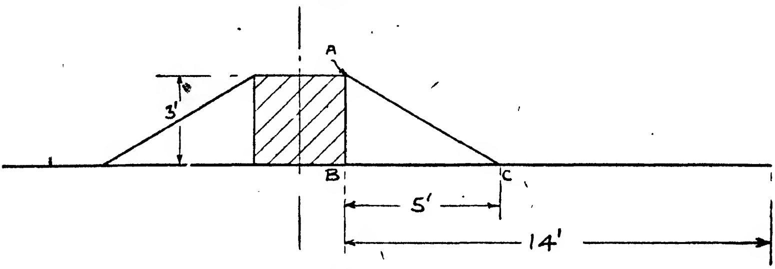
47. Show by means of a sketch the distribution of tensile and compressive stress due to the pure bending of a beam. Explain how this stress distribution shows the advantage of putting the bulk of the material at a distance from the N.A. in order to obtain a maximum strength/weight ratio.

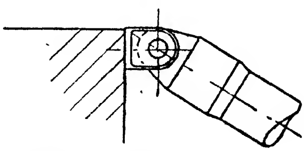
48. The figure shows the arrangement of the front spar and diagonal strut of a semi-cantilever monoplane. If the lift force on the spar is 50 lb. per ft., find—

(*a*) The end loads in *AC* and *BC*.

(*b*) The shear force in the attachment pins at joints *A* and *B*.

(*c*) The diameter of strut *AC* if it is a 17 S.W.G. 40 tons/sq. in. steel tube. Allow an ult. factor of $4\frac{1}{2}$.





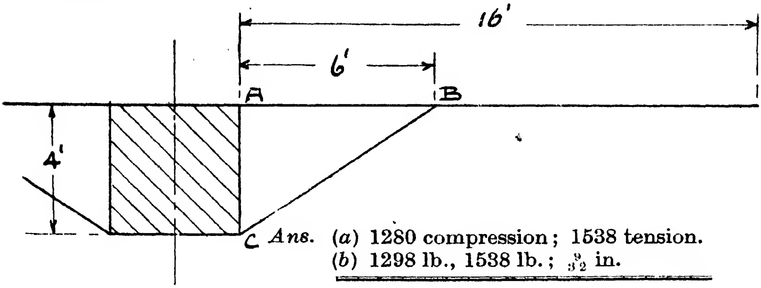
Detail of Joint A

Ans. (*a*) 1904 lb. compression; 1633 lb. tension.
(*b*) 1904 lb.; 1655 lb.
(*c*) 2 in.

49. The figure shows the arrangement of the front spar and diagonal strut of a semi-cantilever monoplane. If the lift force on the spar is 40 lb./ft., find—

(a) The end loads in AB and BC.

(b) The shear force in the pins at A and B and their diameters if they are in double shear. Allow a load factor of 7 and a stress of 40 tons/sq. in.



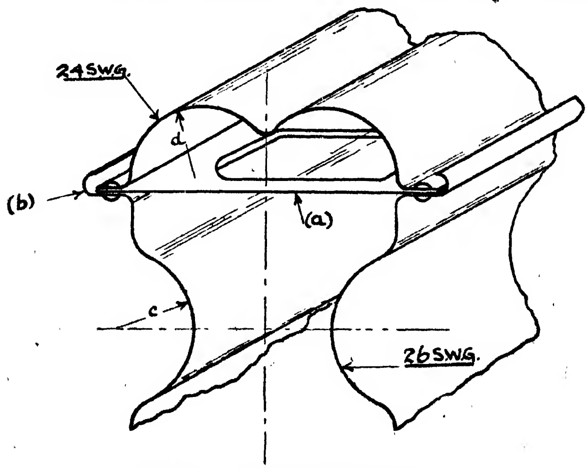
Ans. (a) 1280 compression; 1538 tension.
(b) 1298 lb., 1538 lb.; $\frac{9}{32}$ in.

SKETCH FOR PROBLEM NO. 49

50. Two spars of identical shape, thickness, and length, carry the same uniformly distributed load and compressive end load. One spar is of duralumin and the other of steel. Which will be subjected to the greater stress? *Ans.* Duralumin spar.

51. For the type of steel spar shown in the sketch, state—

(i) Why the member (a) is introduced.
(ii) Why there is a small radius at (b).
(iii) Why the flange is thicker than the web.
(iv) Why the radius/thickness ratio may be greater at (c) than at (d).



SKETCH FOR PROBLEM NO. 51

52. (*a*) What do you understand by elastic stability?

(*b*) State any three rules which have to be considered in the design of a metal spar.

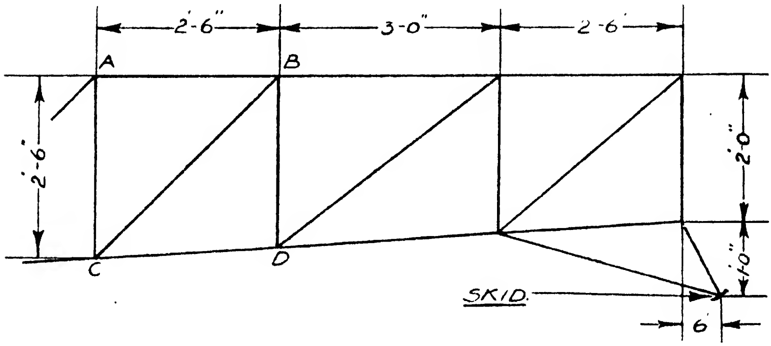
53. A solid shaft, 3 in. diameter, transmits a torque of 4500 ft.-lb. It is replaced by a similar shaft of $3\frac{1}{2}$ in. diameter. If the maximum stress remains the same, what torque will it now transmit?

Ans. 7146 ft.-lb.

54. An aeroplane weighing 4800 lb. is resting on the ground. In this position the horizontal distances of the C.G. and the tail skid from the wheels are 1 ft. 4 in. and 16 ft. respectively. The arrangement of the rear end of the fuselage is shown diagrammatically in the figure. Find—

(*a*) The reaction on the tail skid.

(*b*) The magnitude and nature of the load in members *AB* and *CD*, if the top longeron sloped downwards and rearwards 10° when resting on the ground. (Neglect the weight of the members.)



Ans. (*a*) 400 lb.
(*b*) 663 lb. compression; 461 lb. tension.

55. A cantilever monoplane has a semi-span of 20 ft. and a chord of 6 ft. for the whole span. The front and rear spars are 12 in. and 4 ft. back from the L.E. respectively, and the centre of pressure is one-third of the chord back from the L.E. The total weight of the aircraft is 20,000 lb. and of the wings 2500 lb.

(*a*) If the load distribution is as Fig. 64, Chapter VII, plot a load curve for the front spar.

(*b*) If the front spar is attached to the fuselage by two bolts 10 in. apart, find the horizontal reactions in these bolts.

Ans. 63,000 lb.

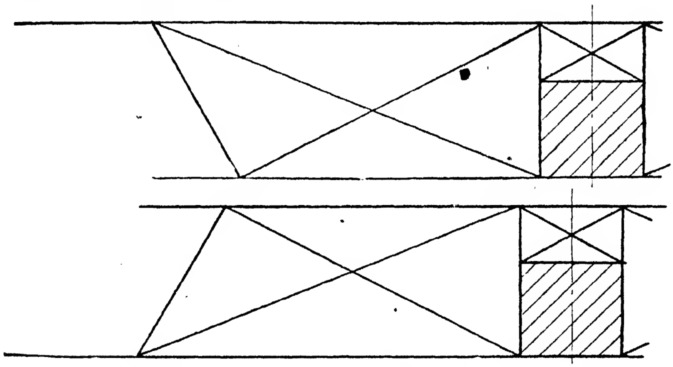
56. What must be the modulus of section of the front spar, in Example 55, at the root end, if the ultimate load factor for the condition

considered is 6, and the material has a yield stress of 60 tons per sq. in. (Neglect end load due to drag bracing).

Ans. $2 \cdot 2$ in.^{3}

57. Make a sketch of a typical two-bay biplane truss, and on it indicate the types of loading on the various members when the aeroplane is in (*a*) normal horizontal flight, (*b*) landing.

58. Discuss the relative strength/weight ratios of the two biplane trusses shown in the sketch.



59. (i) Why is the inner bay of a two-bay biplane usually made shorter than the outer?

(ii) What is the effect on the strength/weight ratio of increasing (*a*) the stagger, (*b*) the aspect ratio?

60. A total load of 8000 lb. has to be transmitted by duplicated lift wires. Due to faulty rigging, one wire has a length of 12 ft. 6·5 in., and the other 12 ft. 6·3 in., when not subjected to load. If the cross-sectional area of each wire is 0·05 sq. in., find the load taken by each wire.

Ans. 3000 lb., 5000 lb.

61. What is the reason for duplicating flying wires, and what rules have to be obeyed when this is done?

62. How may the lift forces be transmitted in a single bay biplane (such as the Bristol *Bulldog*) when the rear flying wires are broken, and how will it affect the loads in the rest of the wing structure?

63. Sketch the side elevation of a typical fuselage, showing the forces which are likely to act on it during high speed flight.

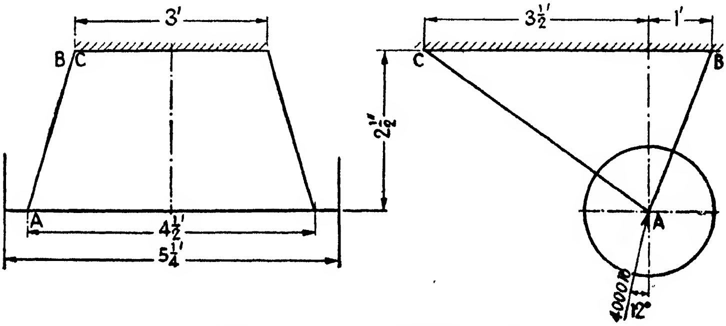
64. What functions has the undercarriage of an aeroplane to perform? Give the nature of the loads to which it may be subjected.

65. Find the maximum ground reaction and the full deflection of an undercarriage for a civil aeroplane weighing 2000 lb. and having a stalling speed of 46 ft./sec. Assume the load-deflection graph is that given in Fig. 74, Chapter VII.

Ans. 6700 lb.; 8·5 in.

66. (*a*) Find the loads in the members of the undercarriage in sketch when it is subjected to a force of 4000 lb. as shown.

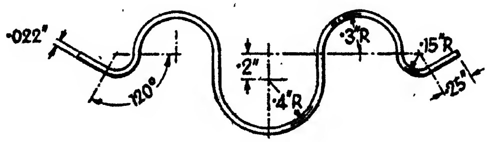
(*b*) Find also the loads when the leg *AB* has been compressed $7\frac{1}{2}$ in. and the external force remains the same.



SKETCH FOR PROBLEM NO. 66

Ans. Axle (*a*) 1140 lb. tension, (*b*) 1550.
AC (*a*) 730 lb., (*b*) 480 lb. compression.
AB (*a*) 3910 lb., (*b*) 4100 lb. compression.

67. Make a dimensional sketch of the shape of the last die required for forming the steel flange shown in the sketch. The steel has a yield stress of 74 tons/sq. in., and $E = 13{,}600$ tons/sq. in.



SKETCH FOR PROBLEM NO. 67

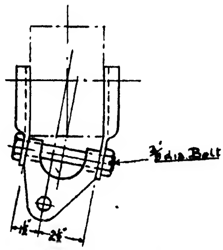
68. Sketch an Armstrong *Atlas* spar, and briefly explain some of the main principles in its design.

69. Sketch a rib of the Warren girder type, distributing the bracing so as to give a high strength/weight ratio. If the bracing is the same section throughout, which member is likely to fail first?

70. The sketch shows a shackle for attaching a lift wire to a spar.

(*a*) *Why is the pin-hole offset in the shackle?*

(*b*) What is the maximum shear stress in the bolt if the load in the wire is 10,400 lb.?



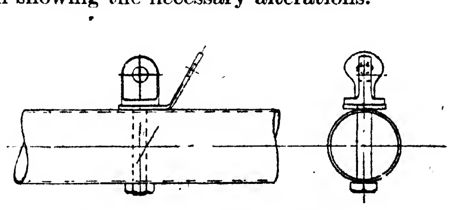
SKETCH FOR PROBLEM NO. 70

Ans. 58,900 lb./sq. in.

71. Design a wiring lug to carry a load of 3450 lb. The maximum stress is not to exceed 26 tons/sq. in. The fork end is 0·2 in. wide and 0·4 in. deep from pin centre to bottom of fork. A ¼ in. diameter pin is used.

72. Sketch a suitable fuselage joint for a wire braced fuselage. The longeron is to be continuous and the struts are to be attached to the fitting with fork sockets.

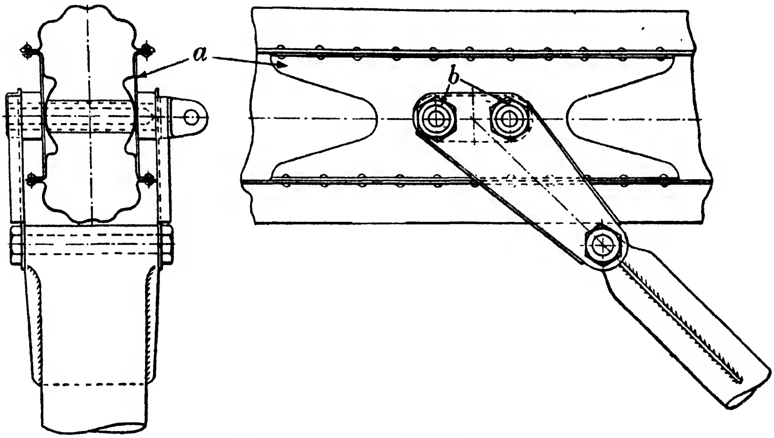
73. What is wrong with the fitting design shown in the sketch? Make a sketch showing the necessary alterations.



SKETCH FOR PROBLEM NO. 73

74. The sketch shows the spar attachment fitting for a lift strut. If the maximum load in the strut is 15,120 lb. tension and the thickness of the spar side plates *a* 0·072 in., find the size of the bolts *b* in order

that they may be of minimum weight. Take the bearing and shear stresses to be 120,000 lb./sq. in. and 54,000 lb./sq. in. respectively.



SKETCH FOR PROBLEM NO. 74

Ans. $\frac{7}{16}$ in. outside diameter.
$\frac{5}{16}$ in. inside diameter.

75. Find the moment of inertia about an axis parallel to and 2 in. below a line joining the centres of the 0·15 in. radii of the flange in Example 67.

Ans. 0·384 in.^{4}

76. The loads in a bracing wire of circular cross-section under three conditions of steady flight are—

C.P. forward = 1800 lb.
C.P. back = 2300 lb.
Nose-dive = 6000 lb.

The worst manoeuvre appropriate to type in C.P. forward attitude is pulling out of a dive at 240 m.p.h. at stalling angle. Stalling speed = 120 m.p.h.

In C.P. back attitude an acceleration of $2g$ is the worst expected acceleration.

No accelerations are obtained during nose-dive.

(*a*) Find the ultimate factors in each case.

(*b*) Find the load a typical wire must stand on test.

(*c*) Find the necessary diameter of the wire if the ultimate stress is 50 tons diag. sq. in. (Proof stress of wire will be greater than 0·75 ultimate stress.)

Ans. 8. 6. 2.
17,280 lb.
0·405 in.

77. (*a*) Explain what you understand by "divergence speed."
(*b*) How may wing flutter be prevented?

78. (*a*) What do you understand by "geodetic construction"?
(*b*) Explain the functions of the skin, longitudinal stringers, and transverse rings of a semi-monocoque fuselage in resisting fracture when it is subjected to bending and shear.

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